Quiz 2.
Please show all of your work.
For this problem, all transistors are minimum length. Technology parameters are given below: $V_{DD} = 1\text{V}$, $V_{Th} = 0.4\text{V}$, $L=30\text{nm}$. Effective mobility is the same for NMOS and PMOS devices. $V_A \to \infty$.

Equations reminder:

$$I_{DS_{sat}} = \frac{W \mu_{eff} C_{ox} E_C L}{L} \frac{(V_{GS} - V_{Th})^2}{(V_{GS} - V_{Th}) + E_C L}$$

$$V_{DS_{sat}} = \frac{(V_{GS} - V_{Th}) E_C L}{(V_{GS} - V_{Th}) + E_C L}$$

$$I_{DS_{lim}} = \frac{\mu C_{ox}}{1 + \left(\frac{V_{DS}}{E_C L}\right)} \frac{W}{L} \left(V_{GS} - V_{Th}\right) V_{DS} - \frac{V_{DS}^2}{2}$$

If an optimally sized NAND2 gate has a logical effort of 1.25, what is the logical effort of an optimally sized NAND3?
(Hint: Recall that the logical effort is the ratio of the gate’s input capacitance to that of an inverter of equivalent drive strength.)

Since the effective mobility is the same for NMOS and PMOS devices, we have $W_p/W_n = 1:1$ for an inverter. Let the sizing of the NMOS in the NAND2 gate be $x$, as shown below:
The logical effort is given by \((1 + x)/2 = 1.25\). We can solve for \(x\) to get \(x = 1.5\). We can solve for \(E_{CL}\) of a single device using the provided \(I_{dsat}\) equation, since we know that the stack has double the effective channel length and that the NAND2 sizing is chosen such that the \(I_{dsat}\) for the NMOS stack remains the same as the \(I_{dsat}\) of an inverter.

Setting \(I_{dsat,inv} = I_{dsat,nand2}\) and canceling common terms, we have the following equation:

\[
\frac{W_{inv}}{(V_{GS} - V_{th} + E_{CL})} = \frac{W_{nand2}}{(V_{GS} - V_{th} + 2E_{CL})}.
\]

We have \(W_{nand2}/W_{inv} = 1.5\) and \(V_{GS} - V_{th} = 0.6\) V. Solving for \(E_{CL}\), we have \(E_{CL} = 0.6\) V.

We can now solve for \(W_{nand3}\) using a similar equation:

\[
W_{inv}/(V_{GS} - V_{th} + E_{CL}) = W_{nand3}/(V_{GS} - V_{th} + 3E_{CL}).
\]

Plugging in values, we get \(W_{nand3} = 2\). This corresponds to a logical effort of \((1 + 2)/2 = 1.5\).