146 04 319

(1) <u>Given:</u> $N=4$, $\begin{cases} f_p = 500\,\text{kHz} & \text{Passband Edge} \\ R_p = 0.3\,\text{dB} & \text{Passband Atten.} \end{cases}$

<u>Needed for Synthesis:</u> N, ω_{-3dB}

<u>Butterworth:</u>

$$|H(\omega)|^2 = \frac{1}{1+\left(\omega/\omega_{-3dB}\right)^{2N}} = \frac{1}{1+\epsilon^2\left(\omega/\omega_p\right)^{2N}}$$

with $\frac{1}{1+\epsilon^2} = \left(10^{-R_p/20}\right)^2$ Passband Atten.

$$\left(\frac{\omega}{\omega_{-3dB}}\right)^{2N} = \epsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}$$

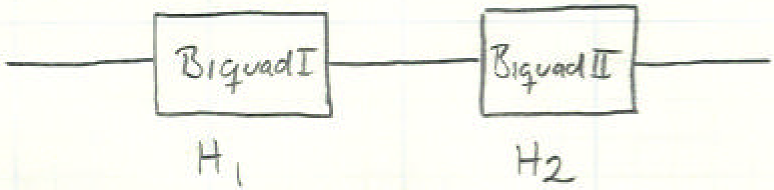
$$\Rightarrow \quad \frac{\omega_{-3dB}}{\omega_p} = \frac{1}{\epsilon^{1/N}}$$

$$\epsilon = \sqrt{10^{R_p/10} - 1} = 0.2674$$

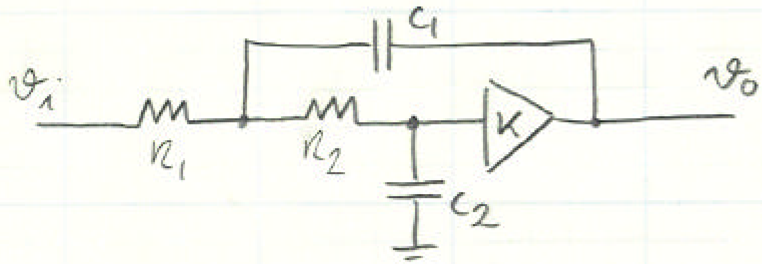
$$\Rightarrow \quad \omega_{-3dB} = 2\pi\, 500\,\text{kHz} \cdot \frac{1}{0.2674^{1/4}}$$

$$\underline{\underline{\omega_{-3dB} = 2\pi \cdot 695.3\ \text{kHz}}}$$

Implementation:



$$H(s) = \frac{G\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$



$$\omega_0 = \frac{1}{\sqrt{R_1C_1R_2C_2}} \qquad G = K \qquad Q = \frac{\omega_0}{\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1-K}{R_2C_2}}$$

from Matlab:

$\omega_{01} = 4.3687$ Mrad/s $\qquad \omega_{02} = 4.3687$ Mrad/s

$Q_1 = 0.5412 \qquad Q_2 = 1.3066$

$G = K = 1$, choose $R_1 = R_2 = R = R_{max} = 100k$ (NOM)

$\Rightarrow Q = \frac{\omega_0 R C_1}{2}$ (note that Q does not change with systematic RC variations!)

$$\boxed{C_1^+ = \frac{2Q}{\omega_0 R^+}}$$ (maximum component values)

$$\omega_0 = \frac{1}{R\sqrt{C_1 C_2}}$$

WORST CASE: $R = R^+ = 1.15\,R$

$C_1 = C_1^+ = 1.1\,C_1$

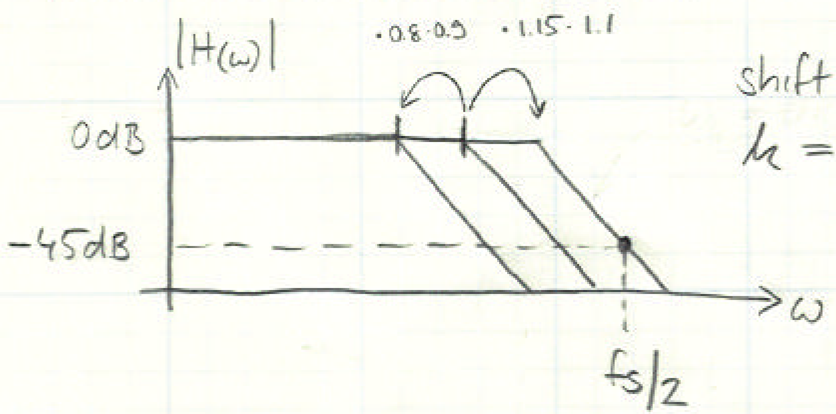
$C_2 = C_2^+ = 1.1\,C_2$

$$\omega_0 = \frac{1}{1.1 \cdot 1.15} \frac{1}{R\sqrt{C_1 C_2}}$$

$$\boxed{C_2^+ = \left(\frac{1}{\omega_0 R^+}\right)^2 \cdot \frac{1}{C_1^+}}$$ (maximum comp. values)

$\Rightarrow$ COMPONENT VALUES SEE MATLAB PRINTOUT

MINIMUM SAMPLING FREQUENCY:



shift by $k = \frac{1}{0.8 \cdot 0.9} \cdot 1.15 \cdot 1.1 = 1.757$

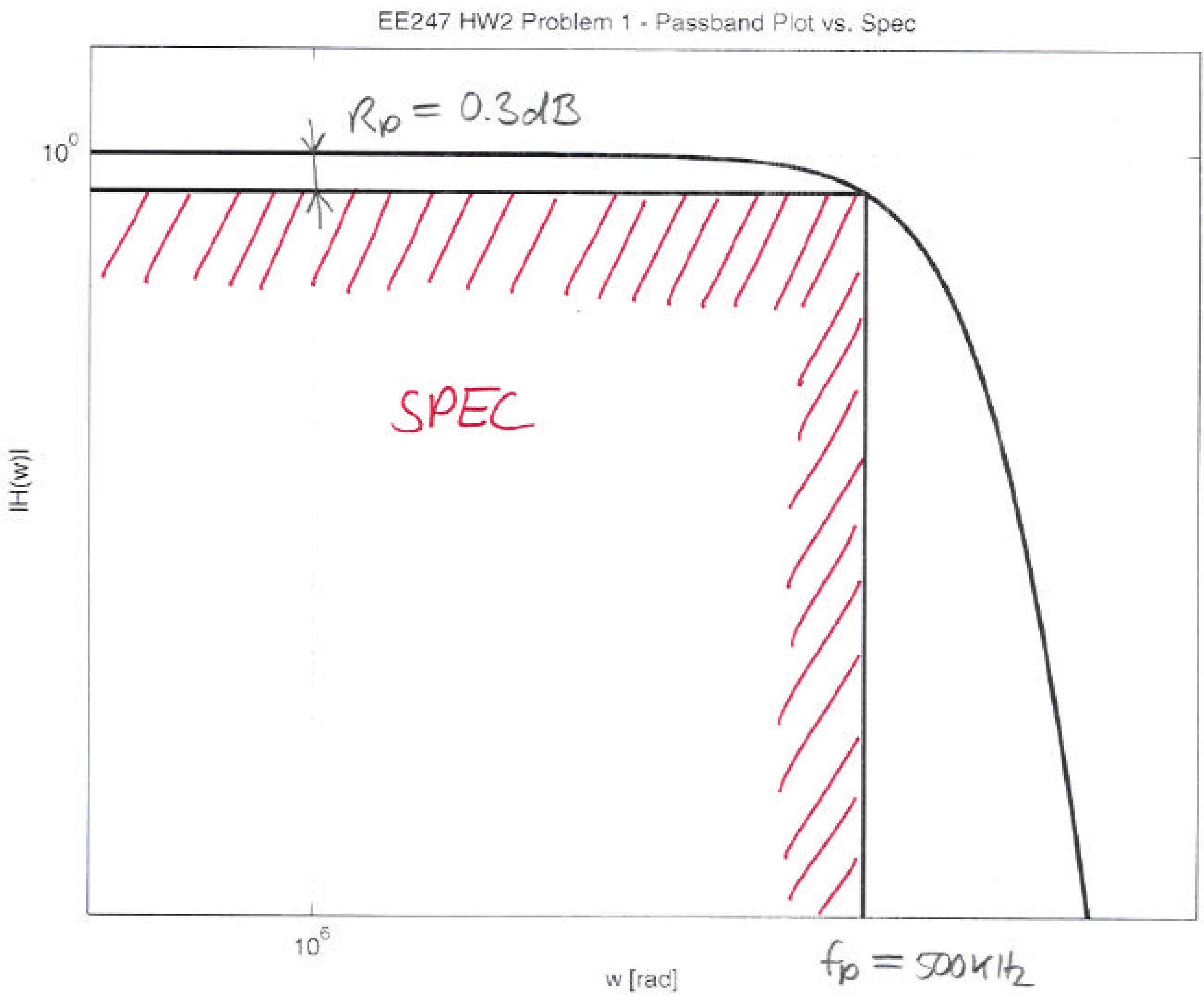
$$|H(f_s/2)|^2 = \frac{1}{1+\left(\frac{f_s}{2f^+_{-3dB}}\right)^{2N}} \cong \left(\frac{2f^+_{-3dB}}{f_s}\right)^{2N}$$

$$f_s \geq \frac{2f^+_{-3dB}}{|H(f_s/2)|^{1/N}} = \frac{2 \cdot 1.757 \cdot 695.3\ \text{kHz}}{10^{-45/(20 \cdot 4)}}$$

$$\boxed{f_s \geq = 8.36\ \text{MHz}}$$

MATLAB OUTPUT:

```
--------------

w01 =
  4.3687e+006

w02 =
  4.3687e+006


Q1 =
    0.5412

Q2 =
    1.3066


COMPONENT VALUES (MAX, NOM, MIN)

R =
  1.0e+005 *
    1.1500    1.0000    0.8500

C11 =
  1.0e-011 *
    0.2154    0.1959    0.1763

C12 =
  1.0e-011 *
    0.5201    0.4728    0.4256

C21 =
  1.0e-011 *
    0.1839    0.1672    0.1505

C22 =
  1.0e-012 *
    0.7617    0.6925    0.6232
```

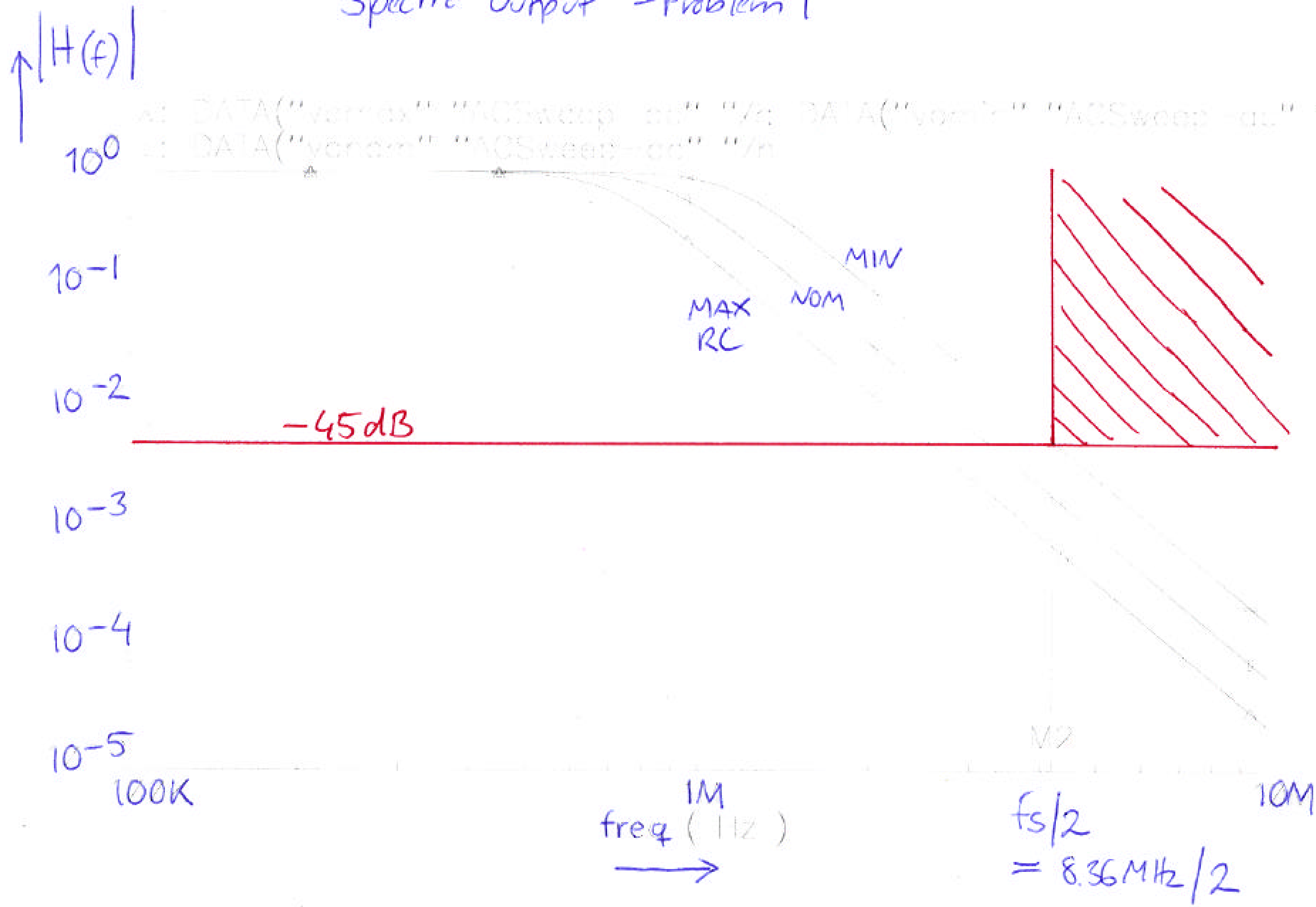


EE247 HW2 Problem 1 - Passband Plot vs. Spec
Rp = 0.3dB
10^0
SPEC
|H(w)|
10^6
w [rad]
fp = 500kHz

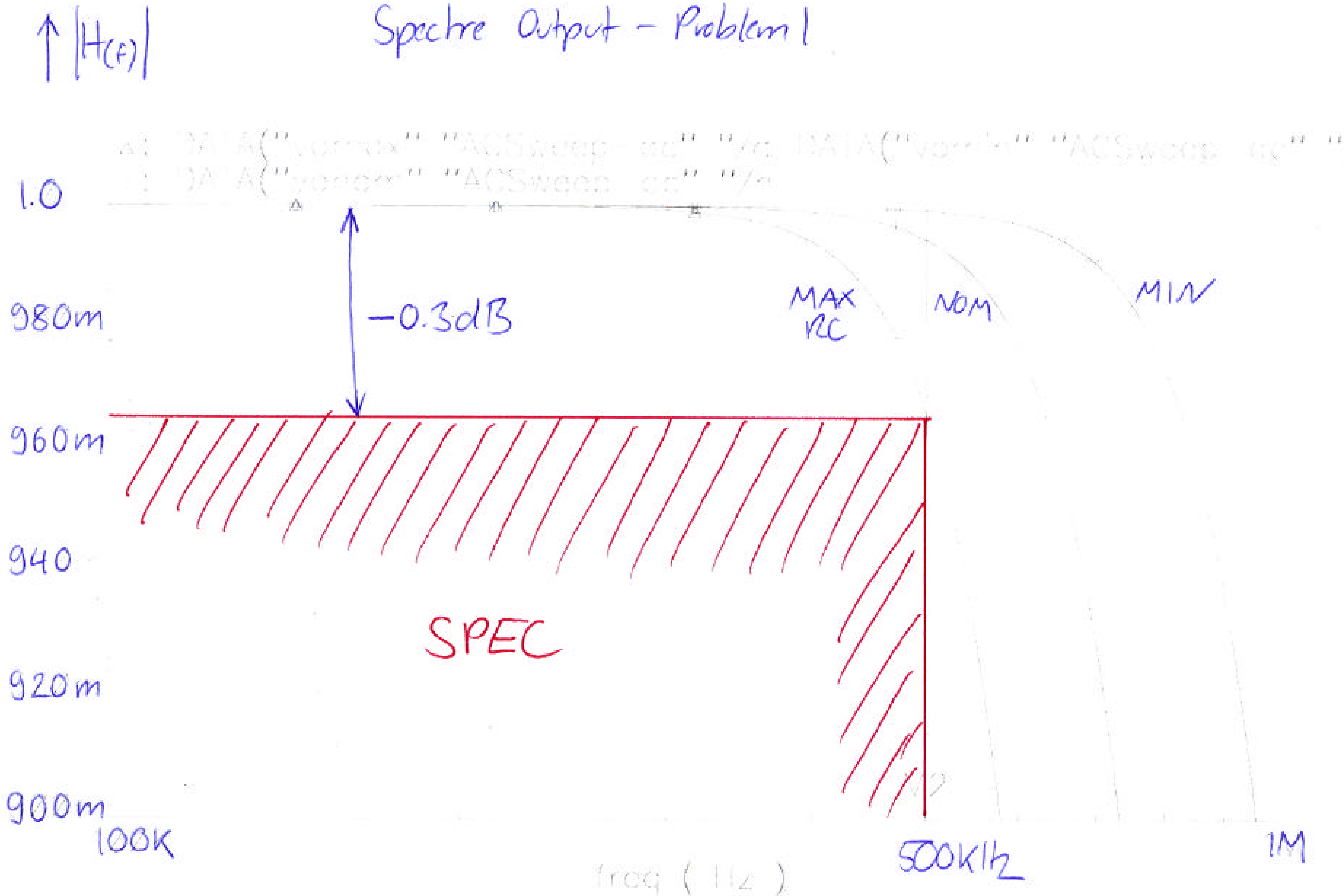
```
% EE247 Homework#2 Problem 1
% Boris Murmann

% filter specification and synthesis
N  = 4;
Wn = 2*pi*695.3e3;       % -3dB frequnency [rad] (hand calculated)
[Z, P, K] = butter(N, Wn, 's');

% Plot frequency response and check vs. spec
w1 = 2*pi*100e3;
w2 = 2*pi*1000e3;
wp = 2*pi*500e3;
Ap= 10^(-0.3/20);
Amin = 0.5;

[NUM, DEN]= zp2tf(Z, P, K);
[H,W]= freqs(NUM, DEN);
loglog( W, abs(H), 'LineWidth',2  );
axis([w1 w2 Amin 1.1]);
line( [w1 wp], [Ap Ap], 'LineWidth',2, 'Color','red');
line( [wp wp], [Ap Amin], 'LineWidth',2, 'Color','red');
title('EE247 HW2 Problem 1 - Passband Plot vs. Spec');
xlabel('w [rad]');
ylabel('|H(w)|');
grid;

% Break up into biquads
[NUM1, DEN1] = zp2tf( [], P(1:2), 1)
[NUM2, DEN2] = zp2tf( [], P(3:4), 1)
w01 = sqrt(DEN1(3))
w02 = sqrt(DEN2(3))
Q1 = w01/DEN1(2)
Q2 = w02/DEN2(2)

% Calculate componenents
Rnom=100e3;
MR=1.15;
MC=1.1;
mR=0.85;
mC=0.9;

% Maximum component values
R(1) = Rnom*MR;
C11(1) = 2*Q1/(w01*R(1));
C12(1) = 2*Q2/(w02*R(1));
C21(1) = (1/C11(1)) * (1/(w01*R(1)))^2;
C22(1) = (1/C12(1)) * (1/(w02*R(1)))^2;

% Nominal component values
R(2) = R(1)/MR;
C11(2) = C11(1)/MC;
C12(2) = C12(1)/MC;
C21(2) = C21(1)/MC;
C22(2) = C22(1)/MC;

% Minimum component values
R(3) = R(2)*mR;
C11(3) = C11(2)*mC;
C12(3) = C12(2)*mC;
C21(3) = C21(2)*mC;
C22(3) = C22(2)*mC;

R
C11
C12
C21
C22
```

```
* EE247 Homework#2 - Problem 1
* Spectre Input
* Boris Murmann

********** Circuit Desription *************

simulator lang=spectre

vin      (vi  0)      vsource mag=1

b1nom    (vi vo1nom)     skbp res=100k cap1=1.959p cap2=1.672p  k=1
b2nom    (vo1nom vonom) skbp res=100k cap1=4.728p cap2=0.6925p k=1

b1min    (vi vo1min)     skbp res=85k  cap1=1.763p cap2=1.505p k=1
b2min    (vo1min vomin) skbp res=85k  cap1=4.256p cap2=0.623p k=1

b1max    (vi vo1max)     skbp res=115k cap1=2.154p cap2=1.839p k=1
b2max    (vo1max vomax) skbp res=115k cap1=5.201p cap2=0.762p k=1

subckt skbp (vi vo)
  parameters cap res cap1 cap2 k
  r1 (vi 1) resistor r=res
  r2 (2  1) resistor r=res
  c1 (1 vo) capacitor c=cap1
  c2 (2  0) capacitor c=cap2
  k1 (vo 0 2 0) vcvs gain=k
ends skbp

********** Control Statements *************

SimOptions options
+        rawfmt=   psfbin
+        gmin=     1E-12
+        reltol=   1E-03
+        vabstol=  1E-06
+        iabstol=  1E-12
+        temp=     27

ACSweep    ac   start=100k stop=10M dec=100
```

Spectre Output - Problem 1



Spectre Output – Problem 1



(2) $H_1(s) = \frac{-K_{oa}}{s+\omega_{oa}}$

$H_2(s) = \frac{K_{2b}s^2 + K_{ob}}{s^2 + \frac{\omega_{ob}}{Q} + \omega_{ob}^2}$ $K_{1b}=0$ (complex conj. zeros)

<u>From Matlab:</u>

$$K \cdot \frac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)\,(s-p_3)}$$

$(s-z_1)(s-z_2)/((s-p_1)(s-p_2))$ ↓ $H_2(s)$; $(s-p_3)$ ↓ $H_1(s)$

- $H_1(s=0) = -\frac{K_{oa}}{\omega_{oa}} \overset{!}{=} -1$ (want unity gain)

  $\omega_{oa} = \frac{-1}{p_3}$; $K_{oa} = \omega_{oa}$

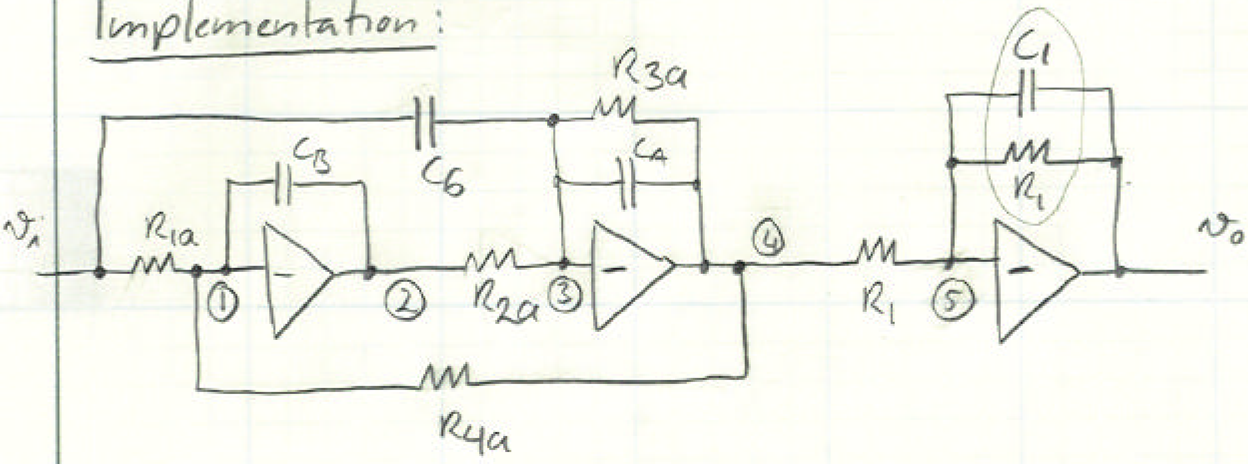
- use zp2tf ($[z_1, z_2]$, $[p_1, p_2]$, K/K_{oa})

  $\Rightarrow \omega_{ob}, Q, K_{ob}, K_{2b}$

→ $\omega_{oa} = 3.29\,\text{Mrad/s} = K_{oa}$] $H_1(s)$

→ $\omega_{ob} = 6.3\,\text{Mrad/s}$
$Q = 2.21$
$K_{ob} = 3.97 \cdot 10^{15}$
$K_{2b} = 0.13\,\frac{1}{\text{rad}^2}$] $H_2(s)$

Implementation:

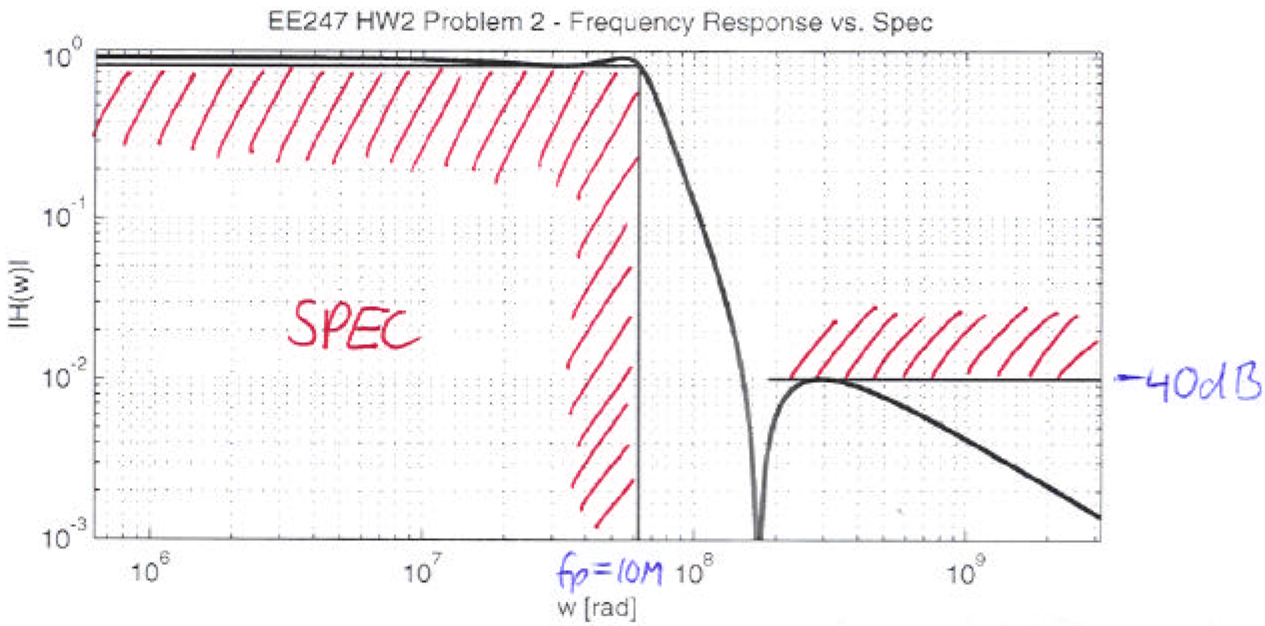


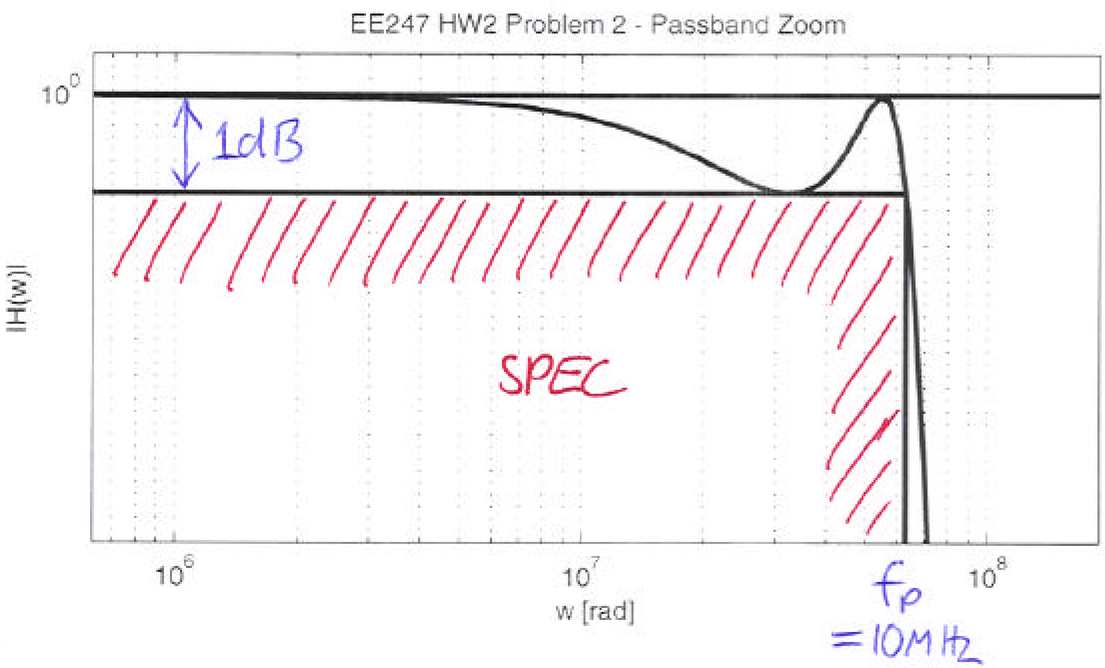
- $C_1 = \frac{1}{\omega_{0a} R_1} = 303.9\,fF$ with $R_1 = R_{max} = 100K$
- → iteravively scale CA, CB until all $R < R_{max}$

using Matlab:

$R_{1a} = 79.37K$	$C_A = 400fF$
$R_{2a} = -39.68K$	$C_B = 200fF$
$R_{3a} = 87.54K$	$C_6 = 52.8\,fF > 50fF$ ✓
$R_{4a} = 79.37K$	

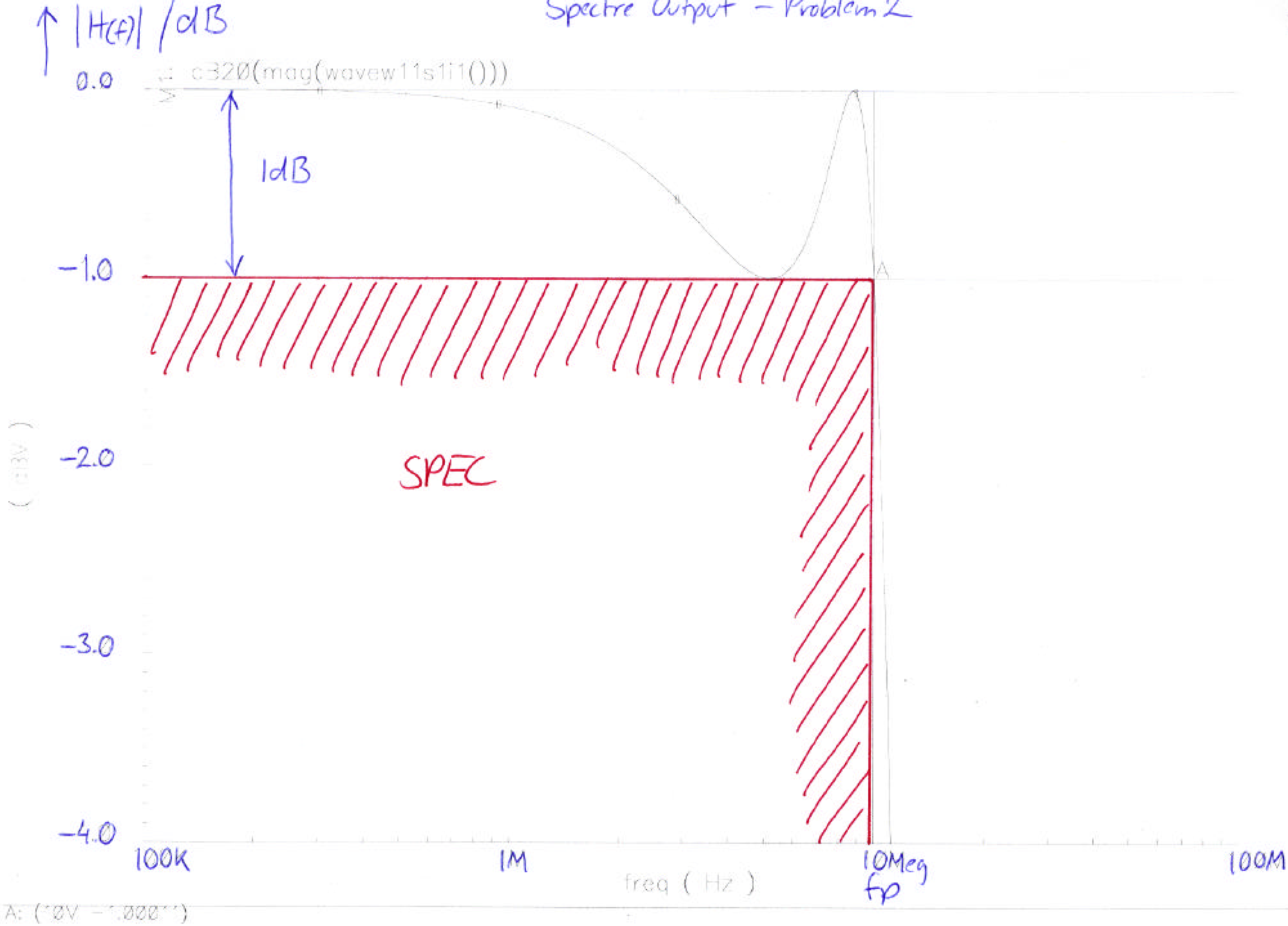
→ Simulation Results see attached


EE247 HW2 Problem 2 - Frequency Response vs. Spec
|H(w)|
w [rad]
SPEC
fp=10M
-40dB

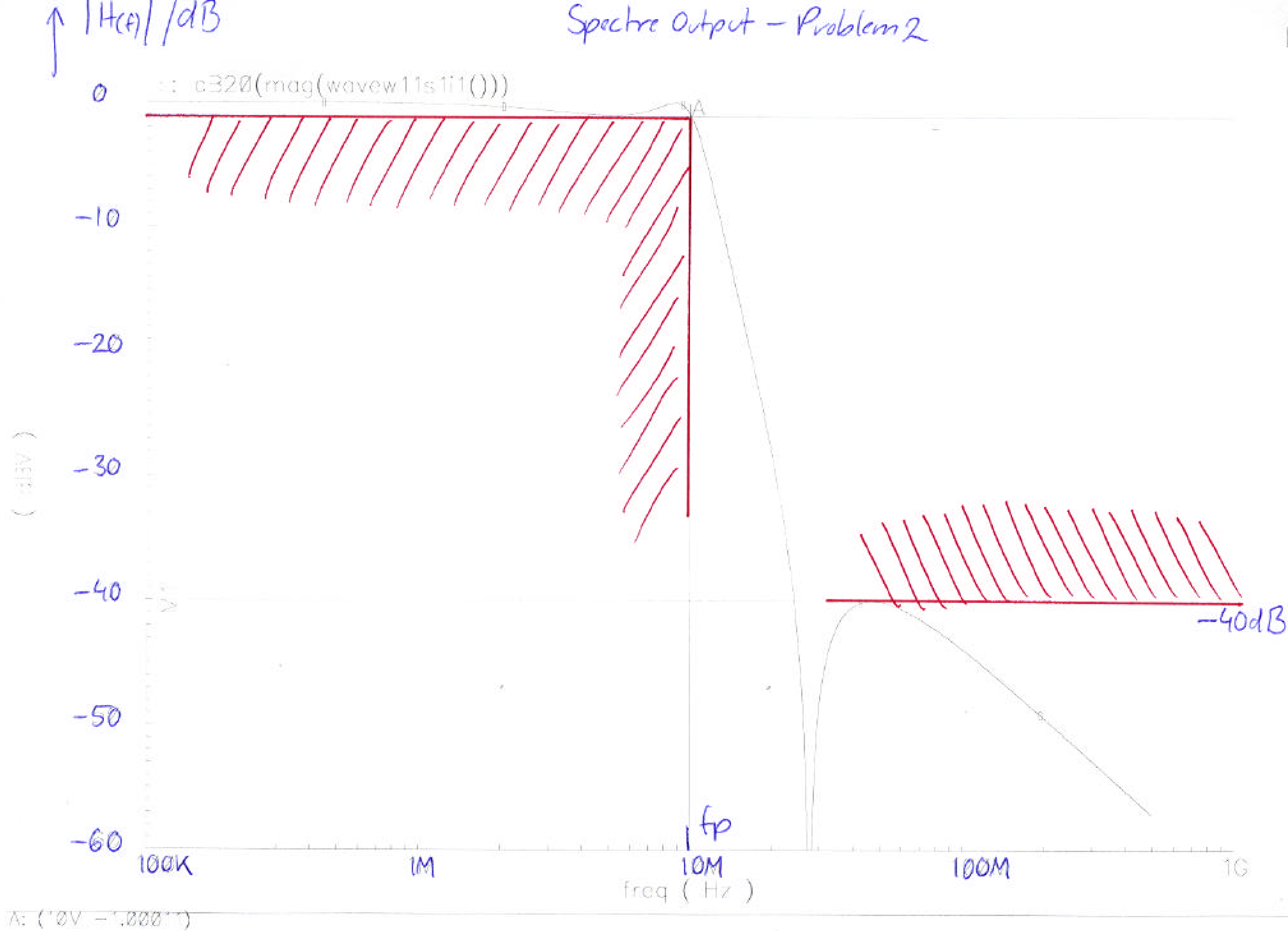

EE247 HW2 Problem 2 - Passband Zoom
|H(w)|
w [rad]
1dB
SPEC
fp
=10MHz

```
% EE247 Homework#2 Problem 2
% Boris Murmann

% filter specification and synthesis
clear;
N  = 3;
Rp = 1;
Rs = 40;
Wn = 2*pi*10e6;      % cutoff frequency [rad]
[Z, P, K] = ellip(N, Rp, Rs, Wn, 's');

% Plot frequency response and check vs. spec
% Plot range
w1 = 2*pi*100e3;
w2 = 2*pi*500e6;
wp = Wn;
ws = 2*pi*30e6;
Ap= 10^(-Rp/20);
As= 10^(-Rs/20);
Amin = 10^((-Rs-20)/20);
[NUM, DEN]= zp2tf(Z, P, K);
[H,W]= freqs(NUM, DEN, w1:(w2-w1)/1000:w2) ;

subplot(2,1,1)
loglog( W, abs(H), 'LineWidth',2  );
axis([w1 w2 Amin 1.1]);
% horizontal passband line
line( [w1 wp], [Ap Ap], 'LineWidth',1, 'Color','red');
%line( [w1 ws], [1  1 ], 'LineWidth',1, 'Color','red');
% horizontal stopband line
line( [ws w2], [As As], 'LineWidth',1, 'Color','red');
% vertical passband line
line( [wp wp], [Ap Amin], 'LineWidth',1, 'Color','red');
% vertical stopband line
%line( [ws ws], [1 As], 'LineWidth',1, 'Color','red');
title('EE247 HW2 Problem 2 - Frequency Response vs. Spec');
xlabel('w [rad]');
ylabel('|H(w)|');
grid;

subplot(2,1,2)
loglog( W, abs(H), 'LineWidth',2  );
axis([w1 ws Ap-0.3 1.05]);
% horizontal passband line
line( [w1 wp], [Ap Ap], 'LineWidth',2, 'Color','red');
line( [w1 ws], [1  1 ], 'LineWidth',2, 'Color','red');
% horizontal stopband line
line( [ws w2], [As As], 'LineWidth',2, 'Color','red');
% vertical passband line
line( [wp wp], [Ap Amin], 'LineWidth',2, 'Color','red');
% vertical stopband line
%line( [ws ws], [1 As], 'LineWidth',2, 'Color','red');
title('EE247 HW2 Problem 2 - Passband Zoom');
xlabel('w [rad]');
ylabel('|H(w)|');
grid;

% Calculate Coefficients

% First order section
% Desired DC Gain is G2=1
G1=1;
w0a=-P(3)
K0a=G1*w0a
% Second order section
% DC Gain is also one, by using K/K0b as the K parameter in zp2tf
[NUM1, DEN1] = zp2tf( [Z(1:2)], P(1:2), K/K0a);
w0b=sqrt(DEN1(3))
```

Spectre Output - Problem 2



Spectre Output – Problem 2



```
* EE247 Homework#2 - Problem 2
* Spectre Input
* Boris Murmann


********** Circuit Desription *************

* component values from matlab
parameters
+R1  = 100k
+C1  = 3.0389e-013
+CA  = 4.0000e-013
+CB  = 2.0000e-013
+R1a = 7.9366e+004
+R2a =-3.9683e+004
+R3a = 8.7539e+004
+R4a = 7.9366e+004
+C6  = 5.2853e-014

vin (vi 0) vsource  mag=1

r1x (vi 1) resistor r=R1a
r2x (2  3) resistor r=R2a
r3x (3  4) resistor r=R3a
r4x (1  4) resistor r=R4a
cax (3  4) capacitor c=CA
cbx (1  2) capacitor c=CB
c6x (vi 3) capacitor c=C6
amp1(2 0 1 0) vcvs gain=-1e6
amp2(4 0 3 0) vcvs gain=-1e6

r11 (4  5) resistor  r=R1
r12 (5 v0) resistor  r=R1
c1x (5 v0) capacitor c=C1
amp3(v0 0 5 0) vcvs gain=-1e6


********** Control Statements *************

SimOptions options
+       rawfmt=   psfbin
+       gmin=     1E-12
+       reltol=   1E-03
+       vabstol=  1E-06
+       iabstol=  1E-12
+       temp=     27

ACSweep    ac  start=100k stop=500Meg dec=100
```

2. Design of a 3rd order elliptic low-pass filter.

(a)

Design Specifications:

Passband:	0-10 MHz
Maximum attenuation in the passband:	1 dB
R_{max}	100kΩ
Filter Type	Elliptic
Gain at f=0	1

By using the MATLAB functions, we can get the transfer function of the desired filter as follow:

$$H(s)=\left(\frac{\omega_C}{s+0.5237\omega_C}\right)\left(\frac{0.0692s^2+0.5265\omega_C^2}{s^2+0.4545\omega_C s+1.0053\omega_C^2}\right)$$

When ω_C is the cut-off frequency of the filter (1dB attenuation), which is desired to be 10MHz

Split it into two filters:

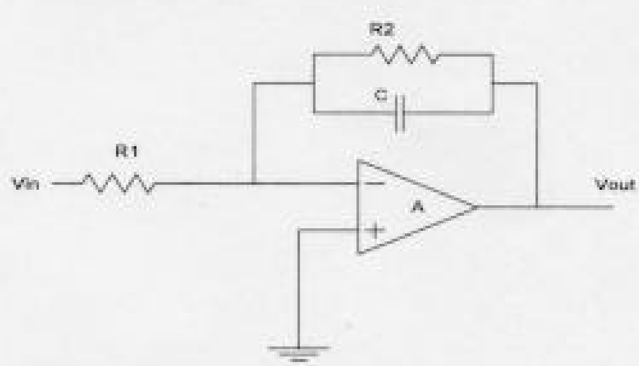
$$H_1(s)=\frac{\omega_C}{s+0.5237\omega_C}$$

$$H_2(s)=\frac{0.0692s^2+0.5265\omega_C^2}{s^2+0.4545\omega_C s+1.0053\omega_C^2}$$

<u>Realizations</u>

<u>$H_1(s)$ realization</u>

We will realize $H_1(s)$ using an opamp with negative feedback, which is shown below:



The transfer function of the filter is:

$$\frac{V_{out}}{V_{in}} = -\frac{\frac{1}{R_1 C}}{s + \frac{1}{R_2 C}} \qquad (1)$$

Ignore the minus sign (just a 180° phase shift) and compare (1) with the $H_1(s)$, we get:

$$\left.\begin{aligned} \frac{1}{R_1 C} &= \omega_C = 62.8 Mrad / Sec \\ \frac{1}{R_2 C} &= 0.5237\omega_C = 32.9 Mrad / Sec \end{aligned}\right\} \qquad (2)$$

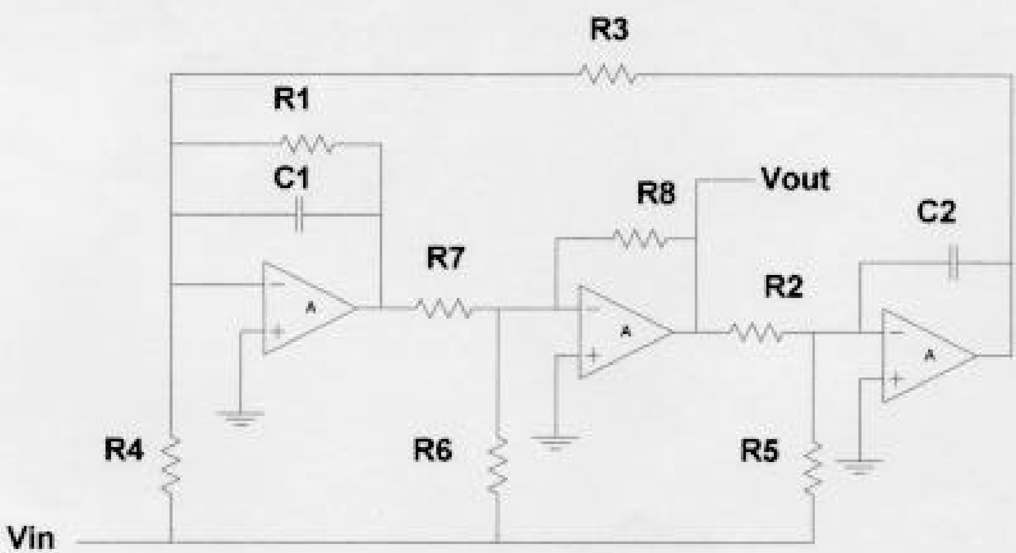
Please note that the maximum realizable resistor is 100kΩ.

Solve (2) with the condition above and try to minimize the size of the capacitor, we have

$R_1 = 52.4k\Omega$ #
$R_2 = 100k\Omega$ #
$C = 304\ fF$ #

<u>$H_2(s)$ realization</u>

We will use the Tow-Thomas biquad to realize $H_2(s)$, a circuit diagram of the biquad is given below:



The transfer function is:

$$\frac{V_{out}}{V_{in}} = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0} \qquad (3)$$

Where:

$$b_0 = \frac{R_8}{R_3 R_5 R_7 C_1 C_2}, \quad b_1 = \frac{1}{R_1 C_1}\left(\frac{R_8}{R_6} - \frac{R_1 R_8}{R_4 R_7}\right), \quad b_2 = \frac{R_8}{R_6}, \quad a_0 = \frac{1}{R_1 C_1}, \quad a_1 = \frac{R_8}{R_2 R_3 R_7 C_1 C_2}$$

Compare (3) with the transfer function of the $H_2(s)$, we get

$b_2 = 0.0692$
$b_1 = 0$
$b_0 = 0.5265\times(2\pi\times 10\text{Mrad/Sec})^2 = 2.079\times 10^3\ (\text{Mad/Sec})^2$
$a_1 = 0.4545\times 2\pi\times 10\text{Mrad/Sec} = 28.56\ \text{Mrad/Sec}$
$a_0 = 1.0053\times(2\pi\times 10\text{Mrad/Sec})^2 = 3.969\times 10^3\ (\text{Mad/Sec})^2$

Now solve the set of the equations above, keep in mind that the C_{min}=50fF, R_{max}=100k, and we try to minimize the total capacitor area.

Select:

$R_1 = R_3 = R_6 = 100k\Omega$, $C_2 = 50fF$ #

So we get:

$$C_1 = \frac{1}{a_1 R_{1,max}} = \frac{1}{(28.56 Mrad/Sec)(100k\Omega)} = 350fF$$ #

$$R_8 = b_2 R_6 = (0.0692)(100k\Omega) = 6.92k\Omega$$ #

$$R_4 R_7 = R_1 R_6 = (100k\Omega)(100k\Omega) \Rightarrow R_4 = R_7 = 100k\Omega$$ #

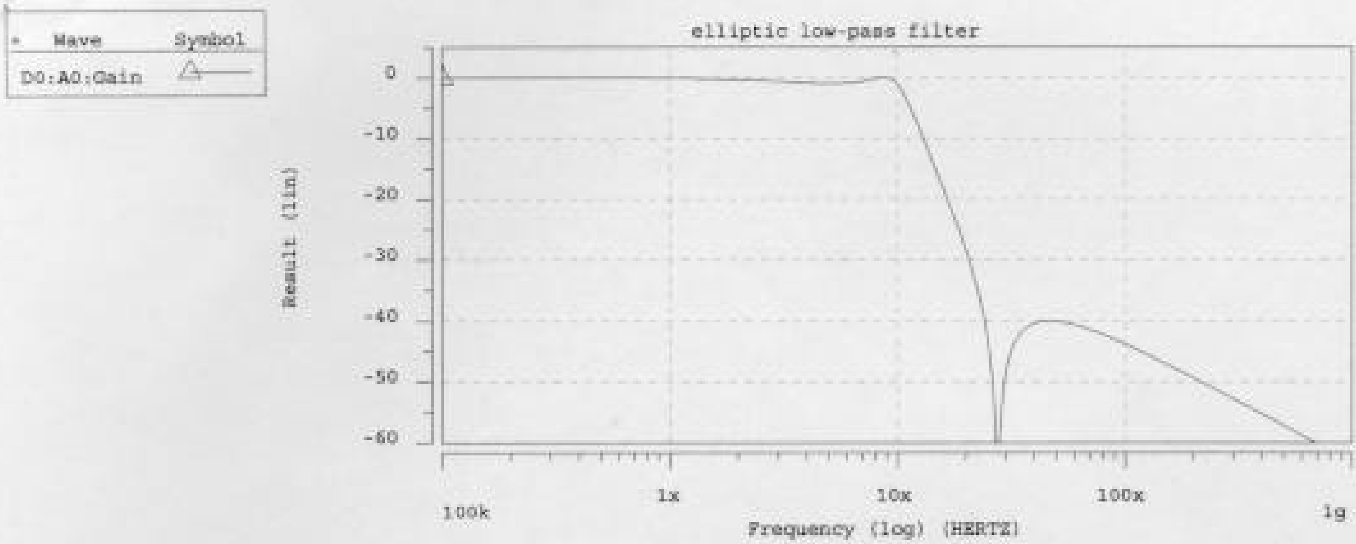
$$R_5 = \frac{R_8}{b_0 R_3 R_7 C_1 C_2} = \frac{6.92k\Omega}{(2.079\times 10^{15} Sec^{-2})(100k\Omega)^2(50fF)(350fF)} = 19.02k\Omega$$ #

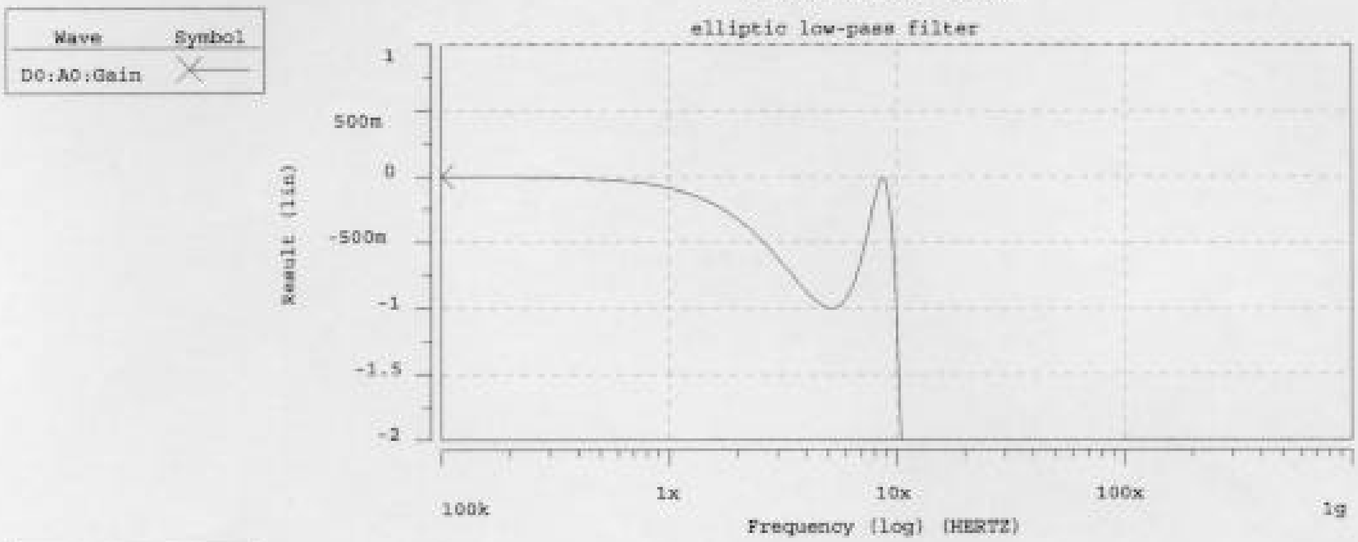
$$R_2 = \frac{b_0}{a_0} R_5 = \frac{2.079}{3.969}(1.902k\Omega) = 9.96k\Omega$$ #

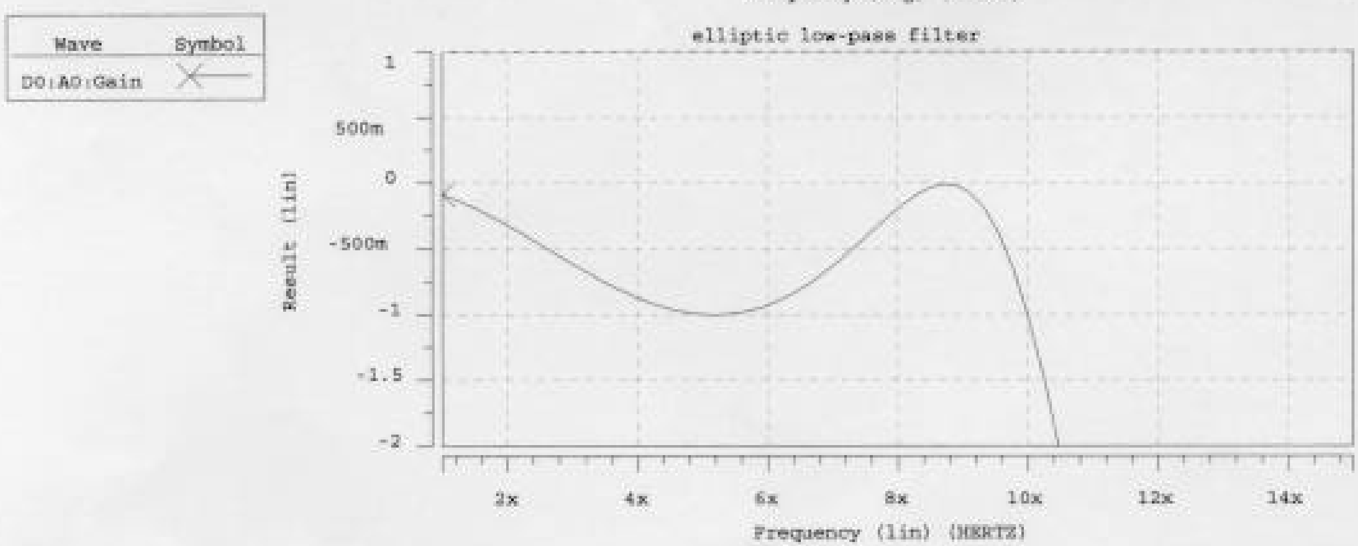
Total Capacitance ($H_1(s)$ and $H_2(s)$)

$$C_{TOT} = 350fF + 50fF + 304fF = 704fF$$ #

SPICE SIMULATION RESULTS



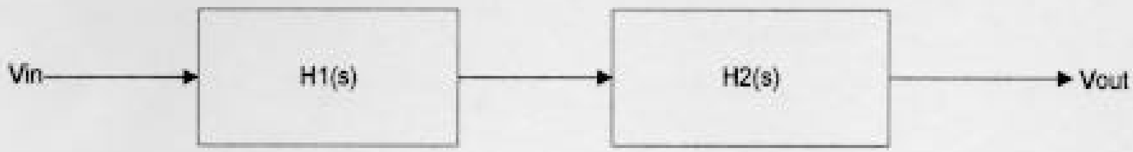




04:57:30 PM Pacific Daylight Time, 09/16/2002

(b)

When we placed the $H_2(s)$ in front of the $H_1(s)$, (see figure below), the total output noise kept increasing as we increased the upper frequency limit for integrating noise.



For f = 0-100MHz, V_n (rms) = 199 μV
For f = 0-1GHz, V_n (rms) = 396 μV
For f = 0-10GHz, V_n (rms) = 1.15 mV (This is already too much) #

The total output noise increased without limit because there is no load capacitance connected to the output node of the $H_2(s)$

So, for the better result (expected), we inverted the order of the first and second order sections. The simulation result in this case is much better. The output noise seems to has its saturation limit.

For f = 0-100MHz, V_n (rms) = 230 μV
For f = 0-1GHz, V_n (rms) = 233 μV
For f = 0-10GHz, V_n (rms) = 233 μV (Saturated) #

(c)

If we try to keep the frequency response of the circuit to be constant, we will have the relationship:

$$V_n(rms) \propto \sqrt{R}$$
$$V_n(rms) \propto \frac{1}{\sqrt{C}}$$

We want to decrease the total output noise from 233 uV to 50 uV, so we have to scale up the sizes of the resistors and scale down the sizes of the capacitors by the factor of:

$$\text{Scale Factor} = \left(\frac{233}{50}\right)^2 = 22 \quad \#$$

<u>For the $H_1(s)$</u>

R_1 = 52.42 kΩ/22 = 38 kΩ, #
R_2 = 100 kΩ/22 = 4.55 kΩ #
C = 304fF×22 = 6.69 pF #

For the $H_2(s)$

$R_1 = R_3 = R_4 = R_6 = R_7 = 100\ k\Omega/22 = 4.55\ k\Omega$
$R_2 = 9.92\ k\Omega/22 = 453\ \Omega$
$R_5 = 19.02\ k\Omega/22 = 864\ \Omega$
$R_8 = 6.92\ k\Omega/22 = 315\ \Omega$
$C_1 = 350fF \times 22 = 7.7\ pF$
$C_2 = 50\ fF \times 22 = 1.1\ pF$ #

Form the SPICE simulations; we get the total out put noise as follow:

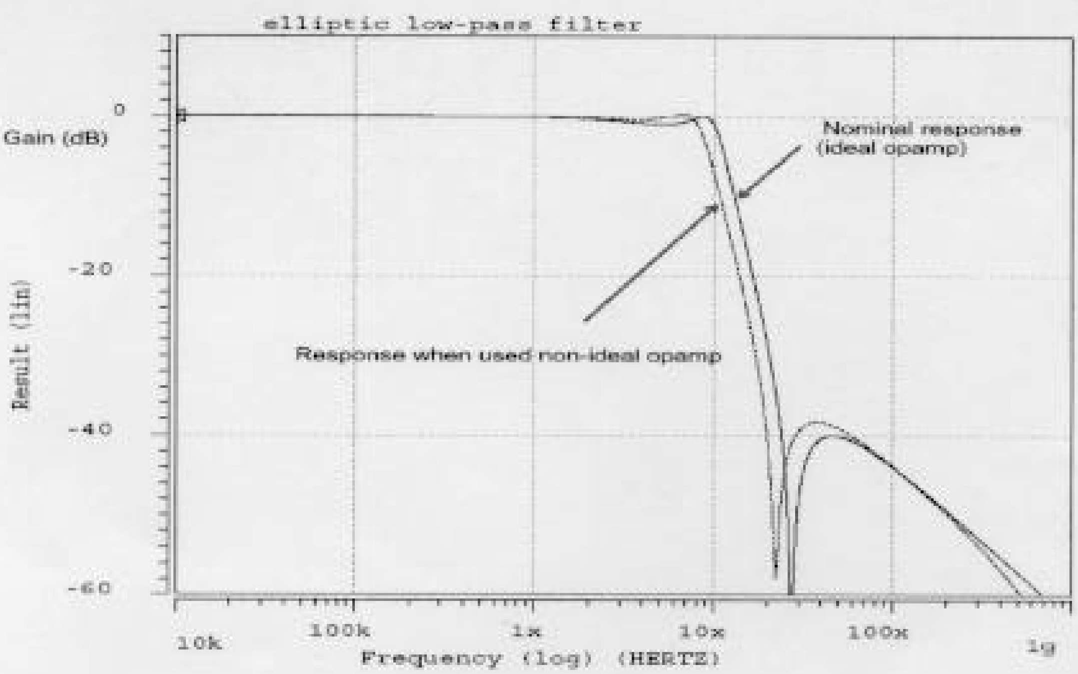
For f = 0-100MHz, V_n (rms) = 49.1 μV
For f = 0-1GHz, V_n (rms) = 49.7 μV
For f = 0-10GHz, V_n (rms) = 49.7 μV (Saturated) #

(d)

Firstly, try opamps with these specifications:

Unity-Gain Bandwidth, f_u	1GHz
Input referred thermal noise, $V_{n,\ in}$	4n $nV/\sqrt{Hz}$

The simulation result shows that the frequency response of the filter has been changed because of the non-ideal opamps (see figure below). We may compensate this change by modifying the component values. However, we will neglect it now and we will do every thing again to see what will happen if we want the better opamp that make the response almost unchanged.



Total output noise obtained from the simulation:

$$V_n\,(rms) = 168\ \mu V$$

The amount of noise contributed by opamp is:

$$V_{opamp}(rms) = \sqrt{(168\mu V)^2 - (50\mu V)^2} = 160\mu V$$

We want the total noise to be 100 μV
So we have to decrease the noise from the opamps to:

$$V_{opamp,new}(rms) = \sqrt{(100\mu V)^2 - (50\mu V)^2} = 86.6\mu V$$

So the input referred noise have to be changed to:

$$V_{n,in}(rms) = \frac{86.6}{160} \times 4nV/\sqrt{Hz} = 2.16nV/\sqrt{Hz} \qquad \#$$

By using the new input referred noise value, the total output noise obtained by SPICE is:

$$V_n\,(rms) = 99.5\ \mu V \qquad \#$$

⇒A part that meets the specifications is MAX4223 from Maxim, which has the characteristics as follow:

Unity-Gain Bandwidth, f_u	1GHz
Input voltage noise at f =10kHz	2n $nV/\sqrt{Hz}$ #

<u>If we want the frequency response to be almost unchanged</u>

⇒From the spice simulations, we need a 10GHz unity-gain opamp bandwidth to guarantee less than 3% changing of the cutoff frequency.

⇒By scaling the value from the previous calculation result, the required input referred voltage noise is

$$V_{n,in} = \left(2.16nV/\sqrt{Hz}\right)\left(\frac{1}{\sqrt{10}}\right) = 0.68nV/\sqrt{Hz} \qquad \#$$

⇒ It is virtually impossible to find any parts that meet the requirements in this case (10GHz Unity-Gain BW, 0.68 $nV/\sqrt{Hz}$ input referred noise).