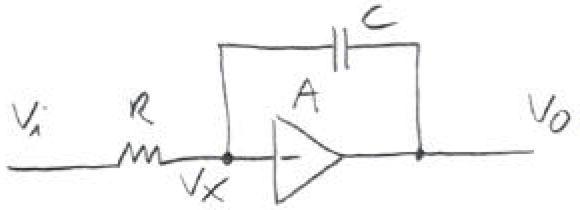
EE247 Homework #4 Boris Murmann

① ⓐ



$\tau = RC$

$A \equiv A_{vo}$

ideal: $\frac{V_o}{V_i} = \frac{-1}{s\tau}$

finite A:

$$\frac{V_i}{R} - \frac{V_x}{R} + V_o sC - V_x sC = 0 \qquad V_x = \frac{-V_o}{A}$$

$$V_i\left(\frac{1}{R}\right) = -V_o\left(\frac{1}{AR} + sC + \frac{sC}{A}\right)$$

$$\frac{V_o}{V_i} = -\frac{1}{\frac{1}{A} + sRC\left(1 + \frac{1}{A}\right)}$$

$$= -\frac{1}{\tau}\,\frac{1}{s\left(1+\frac{1}{A}\right) + \frac{1}{\tau A}}$$

$$\Rightarrow s \longrightarrow s\underbrace{\left(1+\frac{1}{A}\right)}_{B} + \frac{1}{\tau A}$$

$$s \rightarrow sB + \frac{1}{\tau A}$$

Filter: $H\left(sB + \frac{1}{\tau A}\right)\Big|_{s = p_{actual}} \longrightarrow \infty$

$$p_{ideal} = -\alpha_i \pm j\beta_i$$
$$p_{actual} = -\alpha_a \pm j\beta_a$$

$$p_{ideal} = B \cdot p_{actual} + \frac{1}{\tau A}$$

$$\Rightarrow \alpha_i = B\alpha_a + \frac{1}{\tau A}$$
$$\beta_i = B\beta_a$$

$$\Rightarrow \alpha_a = \frac{\alpha_i - \frac{1}{\tau A}}{B} \qquad \Bigg\| \qquad B = 1 + \frac{1}{A}$$
$$\beta_a = \frac{\beta_i}{B} \qquad \Bigg\| \qquad \tau = RC, \quad A \equiv A_{v0}$$

**effect on Q:**

$$Q_a \cong \frac{1}{2}\frac{\beta_a}{\alpha_a} = \frac{1}{2}\frac{\beta_i}{\alpha_i - \frac{1}{\tau A}} = Q_i\left(\frac{1}{1 + \frac{1}{\tau\alpha_i A_{v0}}}\right)$$

with: $\alpha_i = \frac{\omega_0}{2Q_i}$ and $\tau \cong \frac{1}{\omega_0}$

$$\Rightarrow Q_a = Q_i\frac{1}{1 + \frac{2Q}{A}} \cong \boxed{Q_i\left(1 - \frac{2Q}{A}\right)}$$

e.g.: $Q_i = 10$, $A_{v0} = 100$

$$\Rightarrow Q_a \cong 10\left(1 - \frac{20}{100}\right) = \underline{\underline{8}}$$

$$\frac{\Delta Q}{Q} = \underline{\underline{-20\ \%}}$$

$\boxed{\text{effect on } \omega_0 :}$

$$\omega_{oa} = \sqrt{\alpha_a^2 + \beta_a^2}$$

$$\omega_{oa}^2 = \frac{1}{B^2}\left[\left(\alpha_i + \frac{1}{\tau A}\right)^2 + \beta_i^2\right]$$

$$= \frac{1}{B^2}\left[\alpha_i^2 + \beta_i^2 + 2\alpha_i \frac{1}{\tau A} + \left(\frac{1}{\tau A}\right)^2\right]$$

say $\tau \cong \frac{1}{\omega_0}$ ; $\alpha_i = \frac{\omega_{oi}}{2Q_i}$

$$= \frac{1}{\left(1+\frac{1}{A}\right)^2}\left[\omega_{oi}^2 + \frac{\omega_{oi}^2}{AQ_i} + \frac{\omega_{oi}^2}{A^2}\right]$$

$$\cong \omega_{oi}^2 \left(1 - \frac{2}{A}\right)\left(1 + \cancel{\frac{1}{AQ_i}} + \cancel{\frac{1}{A^2}}\right)$$

$A^2 \gg AQ_i \gg \frac{A}{2}$ usually

$$\boxed{\omega_{oa} \cong \omega_{oi}\sqrt{1 - \frac{2}{A}}}$$

<u>e.g.:</u> $A = 100$

$$\Rightarrow \frac{\omega_{oa}}{\omega_{oi}} = \sqrt{1 - \frac{2}{100}} = \underline{\underline{0.99}}$$

$$\frac{\Delta\omega_0}{\omega_0} \cong \underline{\underline{-1\%}} \quad \text{(only)}$$

② ⓐ I decided to use the following design flow:

Synthesize w/ Matlab in z-domain

↓

Break up into Biquads using "tf2sos"

↓

Implement using Hi- and Lo Q circuits as given in Gregorian & Temes pp. 280.

→ see attached schematics with design equations

$$F_{pass} = 2 \cdot \frac{1MHz}{10MHz} = 0.2$$

$$F_{stop} = 2 \cdot \frac{1.5MHz}{10MHz} = 0.3$$

$$\Rightarrow \text{ellipord}(\ldots) = \boxed{6}$$

(filter order)

estimate Biquad Qs, use inverse bilin. transform

$$s_p = \frac{2}{T}\left(\frac{z_p - 1}{z_p + 1}\right)$$

$$\Rightarrow \left.\begin{array}{l} Q_1 = 0.61 \\ Q_2 = 0.16 \end{array}\right\} \text{"Low Q"}$$

$$Q_3 = 6.3 \quad \text{"Hi Q"} \quad (>3)$$

ut

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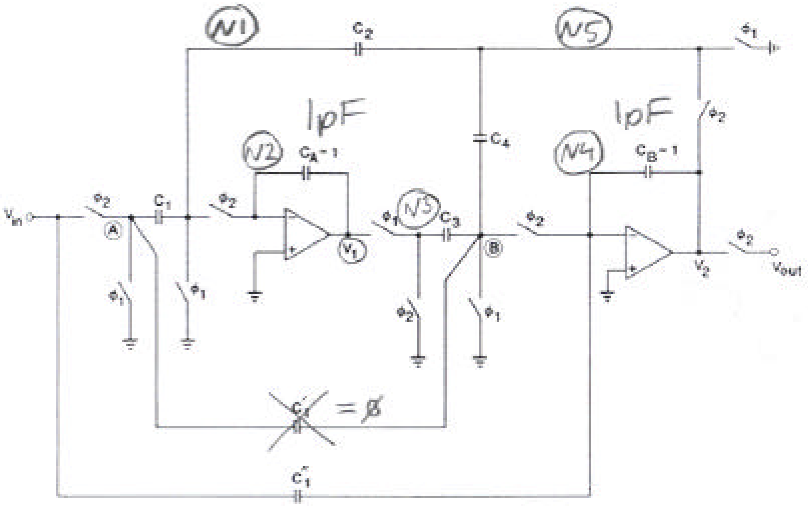


(c)

LoQ

$$H(z) = \frac{a_2 z^2 + a_1 z + a_0}{b_2 z^2 + b_1 z + b_0}$$

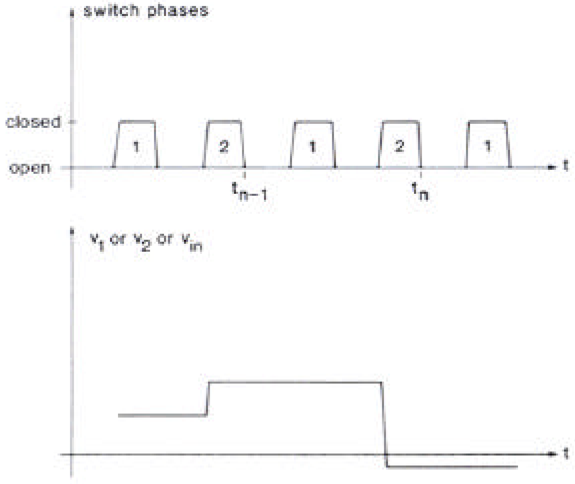
$$C_1'' = a_0$$

$$C_1' = a_2 - a_0$$

$$C_1 = \frac{a_0 + a_1 + a_2}{C_3}$$

$$C_4 = b_2 - 1$$

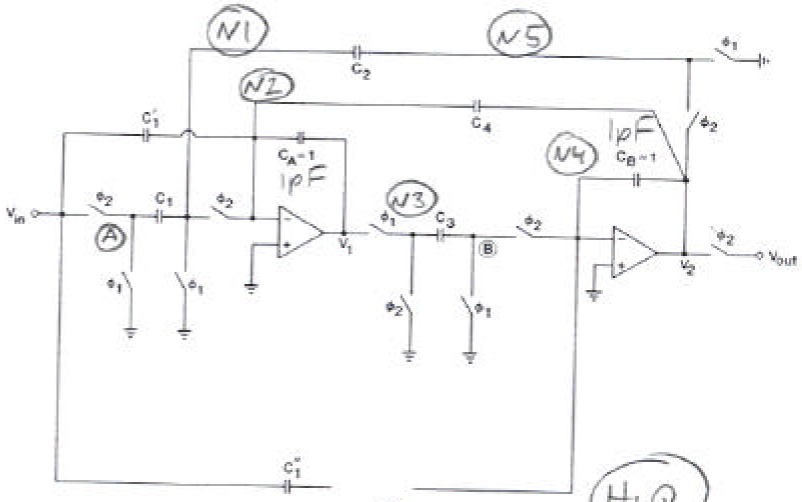
$$C_2 C_3 = b_1 + b_2 + 1$$



(d)

**FIGURE 5.10.** continued.

(Gregorian, Temes)



(c)

H, Q

$$H(z) = \frac{a_2 z^2 + a_1 z + a_0}{b_2 z^2 + b_1 z + b_0}$$

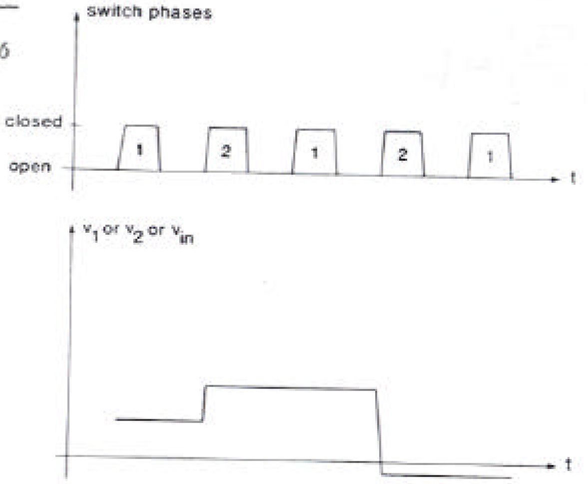
$$C_1'' = \frac{a_2}{b_2}$$

$$C_1' = \frac{a_2 - a_0}{b_2 C_3}$$

$$C_1 = \frac{a_0 + a_1 + a_2}{b_2 C_3}$$

$$C_4 = \frac{1 - 1/b_2}{C_3}$$

$$C_2 C_3 = \frac{1 + b_1 + b_2}{b_2}$$



(d)

**FIGURE 5.13.** continued.

(Gregorian, Temes)

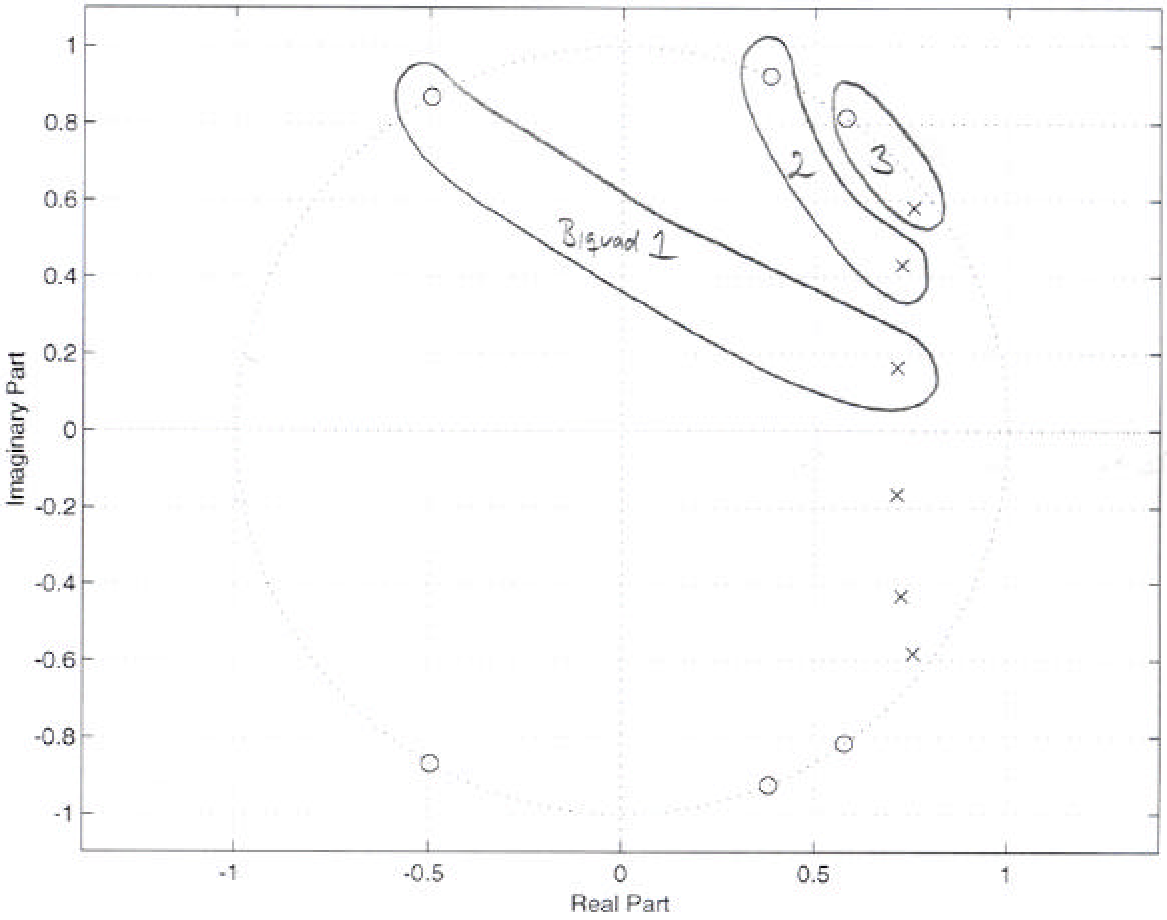
```
2) b) Poles and Zeros
---------------------

Biquad 1:

Z1 =
    -4.959483412658553e-001 +8.683520270003670e-001i
    -4.959483412658553e-001 -8.683520270003670e-001i
P1 =
     7.127856685302593e-001 +1.653876720637911e-001i
     7.127856685302593e-001 -1.653876720637911e-001i

Biquad 2:

Z2 =
     3.825418155817085e-001 +9.239381793883470e-001i
     3.825418155817085e-001 -9.239381793883470e-001i
P2 =
     7.246789558726074e-001 +4.320114746276476e-001i
     7.246789558726074e-001 -4.320114746276476e-001i

Biquad 3:

Z3 =
     5.785179590177820e-001 +8.156696458088302e-001i
     5.785179590177820e-001 -8.156696458088302e-001i
P3 =
     7.553705109318878e-001 +5.808504547226241e-001i
     7.553705109318878e-001 -5.808504547226241e-001i
```

(Grouping done by Matlab "tf2sos"
→ see next page)

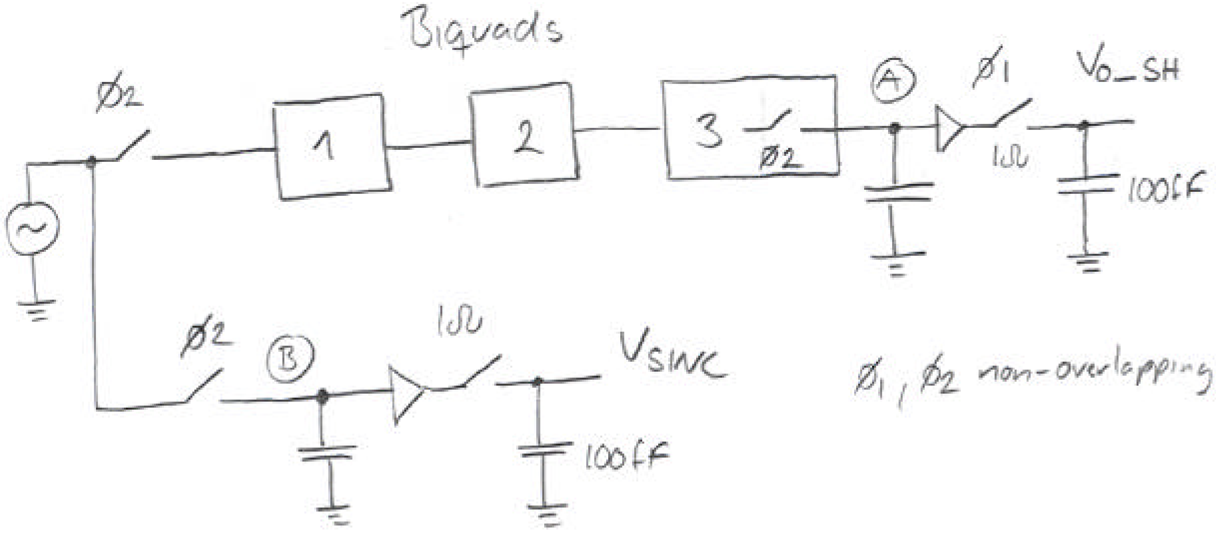
② b)

p/z grouping by Matlab:

("Minimum sensitivity to coefficient tolerances, see e.g. Oppenheim & Schafer)



## ② ⓒ Top Schematic:



$\phi_1, \phi_2$ non-overlapping

→ Ⓐ, Ⓑ are "track and hold nodes", the held value is transferred with $\phi_1$ to form an "ideal" staircase voltage ≡ Dirac sampling with falling edge of $\phi_2$

→ $V_{SINC}$ will be used in the post processor to eliminate sinc distortion.

$$V_{O\_SH}[dB] - V_{SINC}[dB] \Rightarrow \text{actual discrete time response} \;//$$

→ Biquads scaled in Matlab using:

$$\frac{\Sigma a}{\Sigma b} \overset{!}{=} 1 \quad \text{in} \quad \frac{a_2 z^2 + a_1 z + a_0}{b_2 z^2 + b_1 z + b_0} \qquad z \to 1 \text{ @DC}$$

(see Matlab script)

→ Good agreement w/ Spectre simulation, see Plots 1, 2

② ⓓ <u>found:</u> $A_{min} \cong \underline{\underline{1200}}$ for $R_p = 0.12\,dB$, assuming $f_{corner}$ is allowed to shift $\cong \frac{8kHz}{1MHz} = 0.8\%$

→ Stopband attenuation appears to be insensitive to low gain / gain variations.

→ See plots 3, 4

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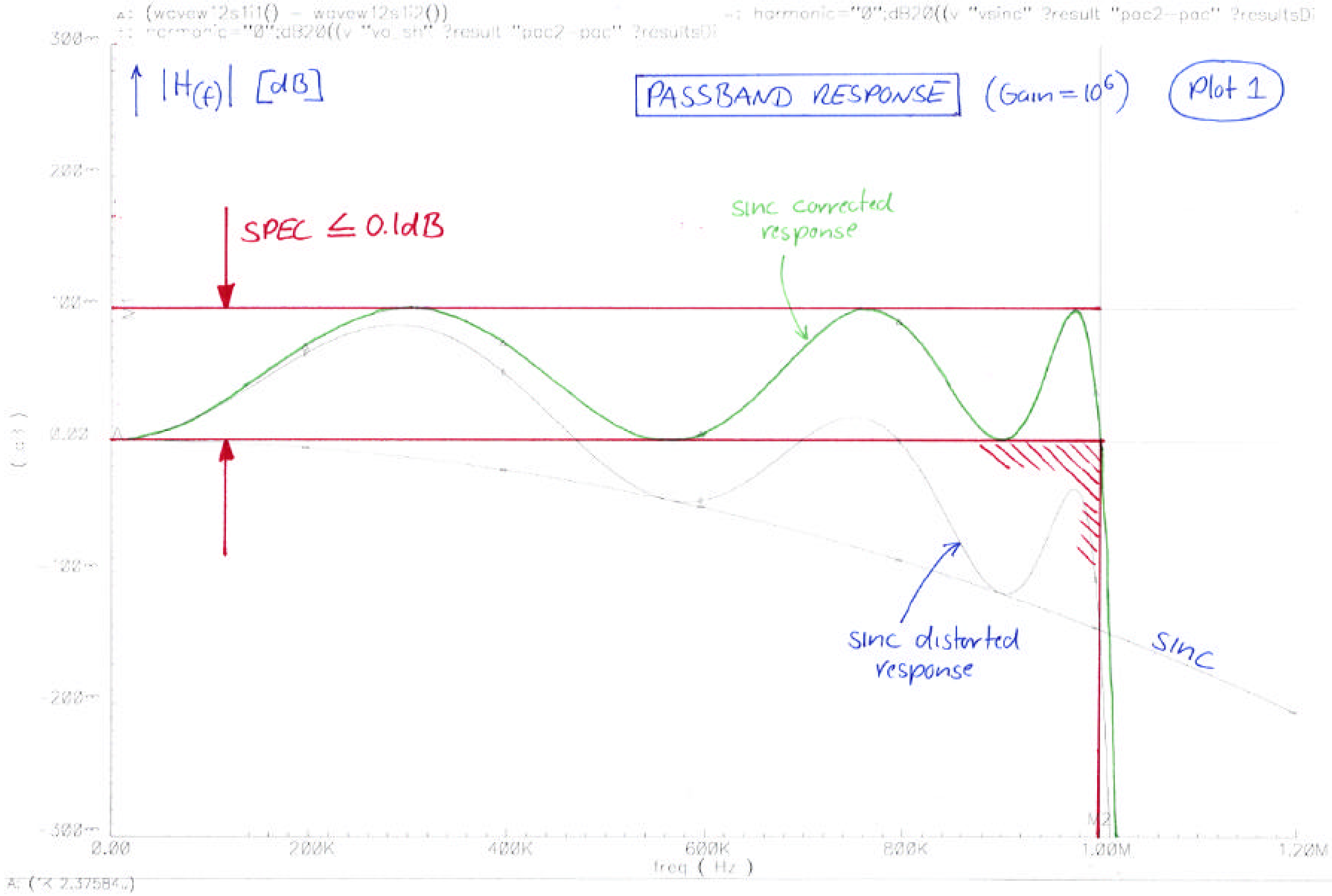
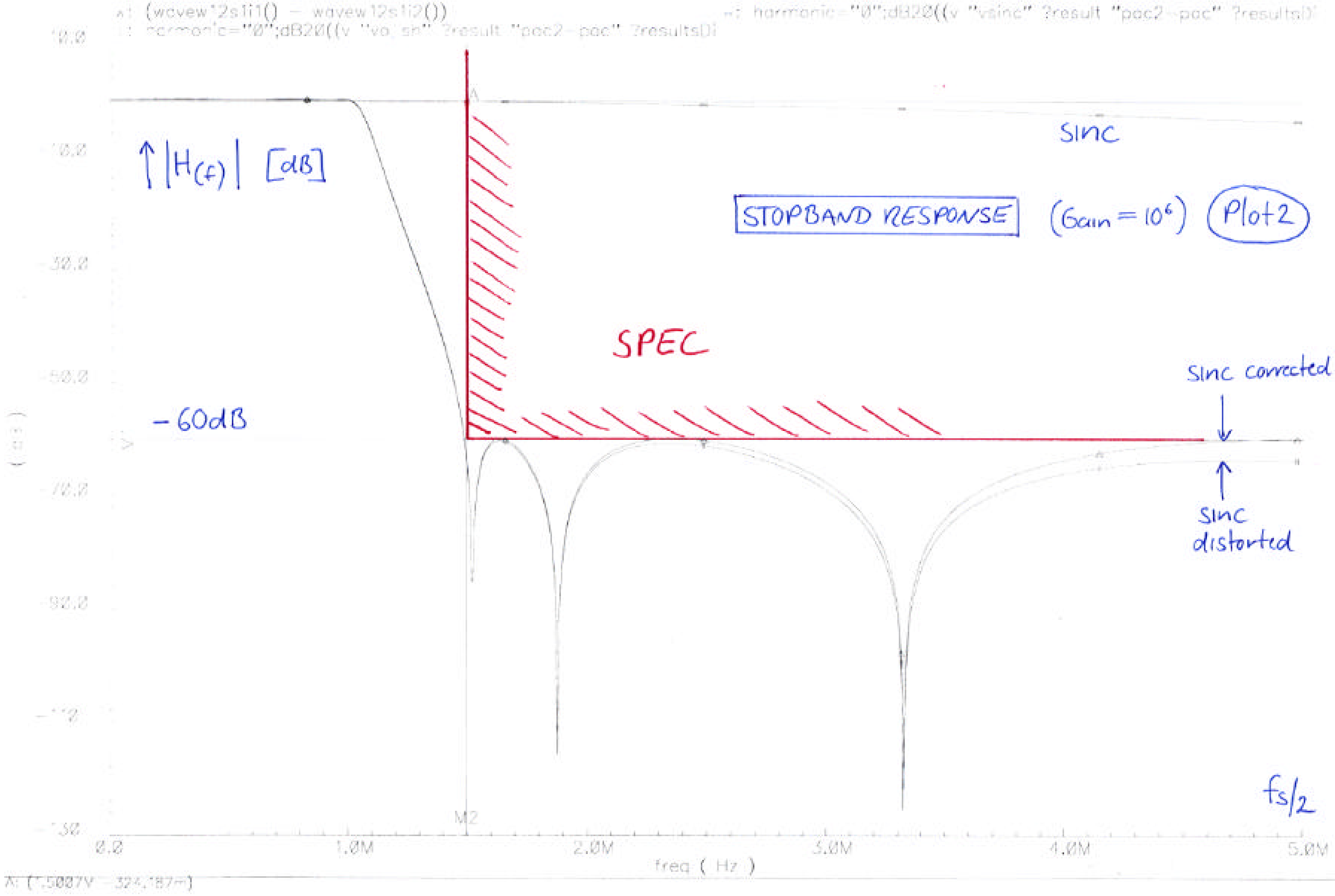
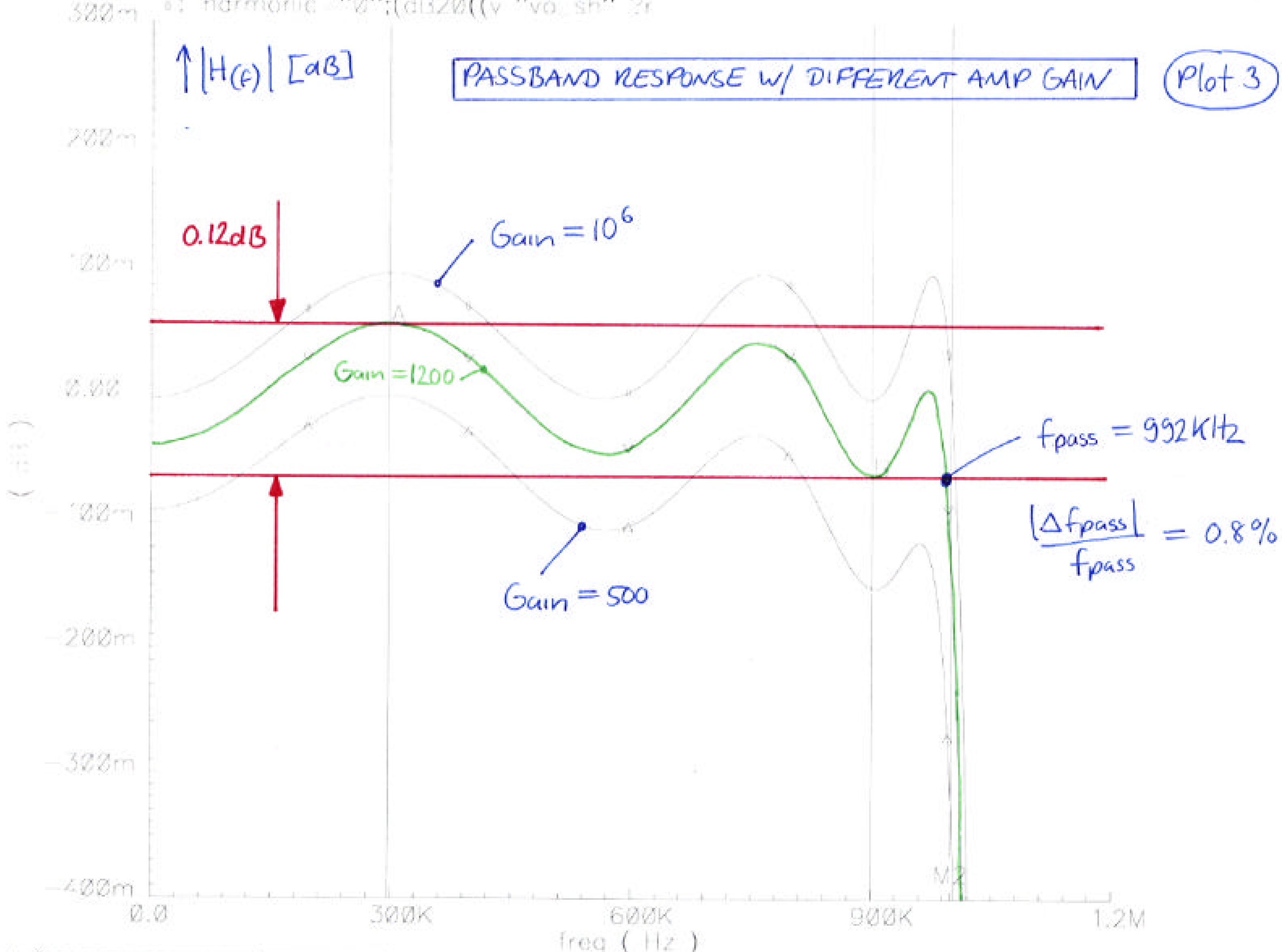


PASSBAND RESPONSE (Gain = $10^6$) Plot 1

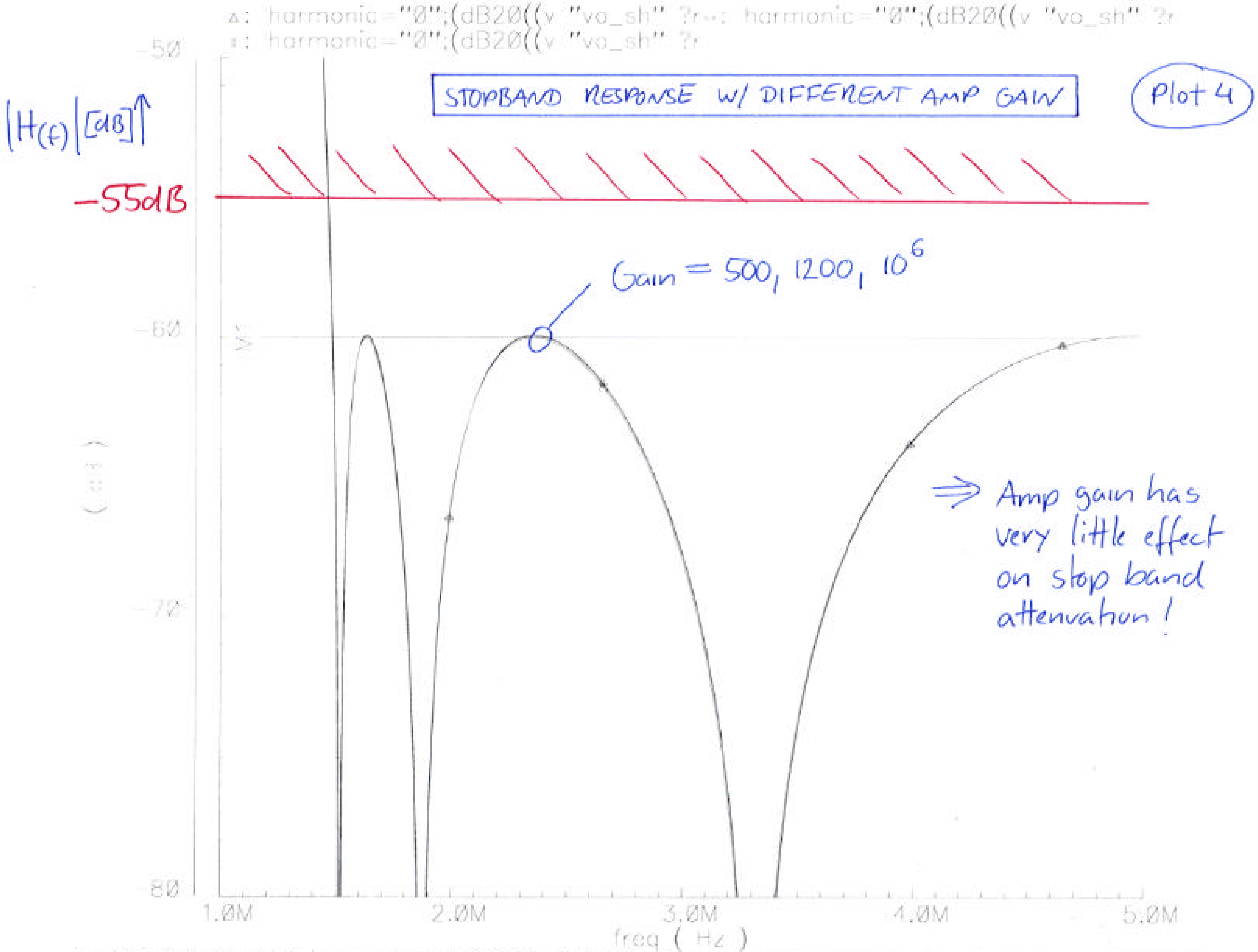


A: (wavew12s1i1() - wavew12s1i2())
harmonic="0";dB20((v "vsinc" ?result "pac2-pac" ?results()
harmonic="0";dB20((v "vo_sh" ?result "pac2-pac" ?results()
10.0
-10.0
-30.0
-50.0
-70.0
-90.0
-110
-130
(dB)
0.0
1.0M
2.0M
3.0M
4.0M
5.0M
freq ( Hz )
M2
A: (1.5007V -324.187m)
↑ |H(f)| [dB]
sinc
STOPBAND RESPONSE
(Gain = 10⁶)
Plot 2
SPEC
-60dB
sinc corrected
sinc distorted
fs/2

```
v: harmonic="0";(dB20((v "vo_sh" ?r∧: harmonic="0";(dB20((v "vo_sh" ?r
s: harmonic="0";(dB20((v "vo_sh" ?r
```



```
A: (299.012K 59.3953m)        delta: (601.808K -121.808m)
B: (900.82K -62.4126m)        slope: -202.403n
```



STOPBAND RESPONSE W/ DIFFERENT AMP GAIN

Plot 4

⇒ Amp gain has very little effect on stop band attenuation!

```
A: (300.94K 67.5233m)          delta: (599.88K -117.389m)
B: (900.82K -49.8657m)         slope: -195.687n
```

Matlab Script ①

```
% EE247 Homework #4 Boris Murmann

% filter specification
clear;
Wpass = 2*pi*1e6;       % passband edge [rad]
Wstop = 2*pi*1.5e6;     % start of stopband [rad]
Rp    = 0.1;            % passband ripple (dB)
Rs    = 60;             % stopband attenuation (dB)

% sampling frequency and normalized band specifications
Wsamp = 2*pi*10e6;
Wp    = 2*Wpass/Wsamp;
Ws    = 2*Wstop/Wsamp;

% discrete time filter synthesis
[N,Wn] = ellipord(Wp, Ws, Rp, Rs);
[num,den] = ellip(N, Rp, Rs, Wn);
freqz(num,den);

% break up into biquads - matlab provides optimized pole zero grouping
[SOS,G] = tf2sos(num,den)
num1 = SOS(1,1:3);
num2 = SOS(2,1:3);
num3 = SOS(3,1:3);
den1 = SOS(1,4:6);
den2 = SOS(2,4:6);
den3 = SOS(3,4:6);

% calculate Q of continous time equivalent sections
[Z1, P1, K1] = tf2zp( num1, den1 );
[Z2, P2, K2] = tf2zp( num2, den2 );
[Z3, P3, K3] = tf2zp( num3, den3 );
s1 = (Wsamp/pi) * (P1(1)-1) / (P1(1)+1);
s2 = (Wsamp/pi) * (P2(1)-1) / (P2(1)+1);
s3 = (Wsamp/pi) * (P3(1)-1) / (P3(1)+1);
Q1 = 0.5*sqrt(1+(imag(s1)/real(s1))^2 )
Q2 = 0.5*sqrt(1+(imag(s2)/real(s2))^2 )
Q3 = 0.5*sqrt(1+(imag(s3)/real(s3))^2 )

% normalize to positive powers of z and divide by "a2" to match (5.27) Gregorian
NUM1 = num1 / den1(3);
NUM2 = num2 / den2(3);
NUM3 = num3 / den3(3);
DEN1 = den1 / den1(3);
DEN2 = den2 / den2(3);
DEN3 = den3 / den3(3);

% scale for unity gain at DC (z=1) for each biquad
k1 = sum(DEN1) / sum(NUM1);
k2 = sum(DEN2) / sum(NUM2);
k3 = sum(DEN3) / sum(NUM3);
NUM1 = k1 * NUM1;
NUM2 = k2 * NUM2;
NUM3 = k3 * NUM3;

% calculate component values, for Q>3 use hi Q circuit
C0=1e-12;
if Q1>3
   Caps1 = HiQ(NUM1, DEN1, C0)
else
   Caps1 = LoQ(NUM1, DEN1, C0)
end
if Q2>3
   Caps2 = HiQ(NUM2, DEN2, C0)
else
```

②

```
   Caps2 = LoQ(NUM2, DEN2, C0)
end
if Q3>3
   Caps3 = HiQ(NUM3, DEN3, C0)
else
   Caps3 = LoQ(NUM3, DEN3, C0)
end
```

```
% Lo Q (Q<3) Gregorian p.286
function  Caps = LoQ(NUM, DEN, C0)
a2 = NUM(1);
a1 = NUM(2);
a0 = NUM(3);
b2 = DEN(1);
b1 = DEN(2);
b0 = DEN(3);
C2 = sqrt(b1+b2+1);
C3 = sqrt(b1+b2+1);
C1x = a2-a0;
C1xx = a0;
C1 = (a0+a1+a2)/C3;
C4 = b2-1;
Caps = C0*[C1,C1x,C1xx,C2,C3,C4];
```

```
% Hi Q (Q>3) Gregorian p.291
function Caps = HiQ(NUM, DEN, C0)
a2 = NUM(1);
a1 = NUM(2);
a0 = NUM(3);
b2 = DEN(1);
b1 = DEN(2);
b0 = DEN(3);
C2 = sqrt( (1+b1+b2)/b2 );
C3 = sqrt( (1+b1+b2)/b2 );
C1x = (a2-a0) / (b2*C3);
C1xx = a2/b2;
C1 = (a0+a1+a2)/ (b2*C3);
C4 = (1 - (1/b2)) / C3;
Caps = C0*[C1,C1x,C1xx,C2,C3,C4];
```

Spectre Netlist

```
*** EE247 HW #4 Boris Murmann

simulator lang=spectre
parameters fs=10Meg  c0=1p Ron=1 AmpGain=1e6
//
*** first low Q section
+c11=    4.529440715720658e-013
+c1x1=   4.163336342344337e-029   // =zero
+c1xx1=  6.857132907366235e-014
+c21=    4.529440715720658e-013
+c31=    4.529440715720658e-013
+c41=    8.677048917763188e-013
//
*** second low Q section
+c12=    6.072036269121990e-013
+c1x2=   -8.881784197001252e-028  //=zero
+c1xx2=  2.985596869872886e-013
+c22=    6.072036269121991e-013
+c32=    6.072036269121991e-013
+c42=    4.049018365178336e-013
//
*** hi Q section
+c13=    6.302625148882051e-013
+c1x3=   -2.079239384327061e-027  //=zero
+c1xx3=  4.712310360217251e-013
+c23=    6.302625148882049e-013
+c33=    6.302625148882049e-013
+c43=    1.460155701618134e-013

*** stimulus
vin1    (vin      0)  vsource pacmag=1
//vin1    (vin     0)  vsource type=sine ampl=1 freq=fs/10
vphi1   (phi1     0)  vsource type=pulse val0=0 val1=1 period=1/fs
width=0.45/fs rise=.1n fall=.1n delay=0.05/fs
vphi2   (phi2     0)  vsource type=pulse val0=0 val1=1 period=1/fs
width=0.45/fs rise=.1n fall=.1n delay=0.55/fs

*** circuit
b1    (vin  vo1   phi1  phi2)  loq  pc0=c0 pc1=c11 pc1xx=c1xx1 pc2=c21
pc3=c31 pc4=c41
b2    (vo1  vo2   phi1  phi2)  loq  pc0=c0 pc1=c12 pc1xx=c1xx2 pc2=c22
pc3=c32 pc4=c42
b3    (vo2  vout  phi1  phi2)  hiq  pc0=c0 pc1=c13 pc1xx=c1xx3 pc2=c23
pc3=c33 pc4=c43

*** sample and hold output
c1    (vout  0)                capacitor c=100f
buf1  (voutbuf 0       vout  0)  vcvs gain=1
s2      (voutbuf vo_sh  phi1  0)  relay vt1=0 vt2=1 rclosed=1
ropen=100e9
c2    (vo_sh       0)                capacitor c=100f

*** sample and hold input (-> sinc response for correction)
s11     (vin       vth   phi2  0)  relay vt1=0 vt2=1 rclosed=1
ropen=100e9
c11   (vth  0)                 capacitor c=100f
buf11 (vthbuf 0        vth   0)  vcvs gain=1
s21     (vthbuf vsinc   phi1  0)  relay vt1=0 vt2=1 rclosed=1
```

```
ropen=100e9
c21    (vsinc       0)                  capacitor c=100f

*******************************************************************
* Lo Q biquad
subckt loq (vi vos phi1 phi2)
  parameters pc0 pc1 pc1xx pc2 pc3 pc4
  rsim1       (vi    n2     phi2  phi1  a      n1)    rsim_series
Csamp=pc1
  rsim2       (v1    n4     phi1  phi2  n3     b )    rsim_series_inv
Csamp=pc3
  c1xx              (vi    n4)    capacitor   c=pc1xx
  c2          (n1    n5)    capacitor   c=pc2
  c4          (n5    b)     capacitor   c=pc4
  switch1           (n5    0      phi1   0) relay vt1=0 vt2=1 rclosed=Ron
ropen=100e9
  switch2           (n5    vo     phi2   0) relay vt1=0 vt2=1 rclosed=Ron
ropen=100e9
  switch3           (vos          vo     phi2   0) relay vt1=0 vt2=1
rclosed=Ron ropen=100e9
  amp1        (v1    0      0      n2) vcvs gain=AmpGain
  amp2        (vo    0      0      n4) vcvs gain=AmpGain
  ca          (n2    v1)    capacitor   c=pc0
  cb          (n4    vo)    capacitor   c=pc0
ends loq

* Hi Q biquad
subckt hiq (vi vos phi1 phi2)
  parameters pc0 pc1 pc1xx pc2 pc3 pc4
  rsim1       (vi    n2     phi2  phi1  a      n1)    rsim_series
Csamp=pc1
  rsim2       (v1    n4     phi1  phi2  n3     b )    rsim_series_inv
Csamp=pc3
  c1xx              (vi    n4)    capacitor   c=pc1xx
  c2          (n1    n5)    capacitor   c=pc2
  c4          (n2    vo)    capacitor   c=pc4
  switch1           (n5    0      phi1   0) relay vt1=0 vt2=1 rclosed=Ron
ropen=100e9
  switch2           (n5    vo     phi2   0) relay vt1=0 vt2=1 rclosed=Ron
ropen=100e9
  switch3           (vos          vo     phi2   0) relay vt1=0 vt2=1
rclosed=Ron ropen=100e9
  amp1        (v1    0      0      n2) vcvs gain=AmpGain
  amp2        (vo    0      0      n4) vcvs gain=AmpGain
  ca          (n2    v1)    capacitor   c=pc0
  cb          (n4    vo)    capacitor   c=pc0
ends hiq

* simulated resistor with capacitor in series
subckt rsim_series (vl vr phi1 phi2 n1 n2)
  parameters Csamp
  cs      (n1  n2)           capacitor c=Csamp
  s1      (vl  n1  phi1 0 ) relay vt1=0 vt2=1 rclosed=Ron ropen=100e9
  s2      (n1  0   phi2 0 ) relay vt1=0 vt2=1 rclosed=Ron ropen=100e9
  s3      (n2  0   phi2 0 ) relay vt1=0 vt2=1 rclosed=Ron ropen=100e9
  s4      (n2  vr  phi1 0 ) relay vt1=0 vt2=1 rclosed=Ron ropen=100e9
  cp1   (n1  0)            capacitor  c=1f
  cp2   (n2  0)            capacitor  c=1f
```

```
ends rsim_series

* simulated resistor with capacitor in series and charge inversion
subckt rsim_series_inv (vl vr phi1 phi2 n1 n2)
  parameters Csamp
  cs      (n1  n2)            capacitor c=Csamp
  s1      (vl  n1  phi1 0 ) relay vt1=0 vt2=1 rclosed=Ron ropen=100e9
  s2      (n1  0   phi2 0 ) relay vt1=0 vt2=1 rclosed=Ron ropen=100e9
  s3      (n2  0   phi1 0 ) relay vt1=0 vt2=1 rclosed=Ron ropen=100e9
  s4      (n2  vr  phi2 0 ) relay vt1=0 vt2=1 rclosed=Ron ropen=100e9
  cp1   (n1  0)            capacitor  c=1f
  cp2   (n2  0)            capacitor  c=1f
ends rsim_series_inv
*********************************************************************

pss2  pss   fund=fs   maxacfreq=fs/2 tstab=1/fs/4
pac2  pac   start=1k  stop=fs/2 lin=1000
*tran1 tran  stop=10/fs method=gear2only

simOptions options
+         rawfmt=psfbin
+           reltol=1e-5
+           vabstol=1n
+           iabstol=1p

alter1 alter param=AmpGain value=1.2e3
pss3  pss   fund=fs   maxacfreq=fs/2 tstab=1/fs/4
pac3  pac   start=1k  stop=fs/2 lin=1000

alter2 alter param=AmpGain value=0.5e3
pss4  pss   fund=fs   maxacfreq=fs/2 tstab=1/fs/4
pac4  pac   start=1k  stop=fs/2 lin=1000
```