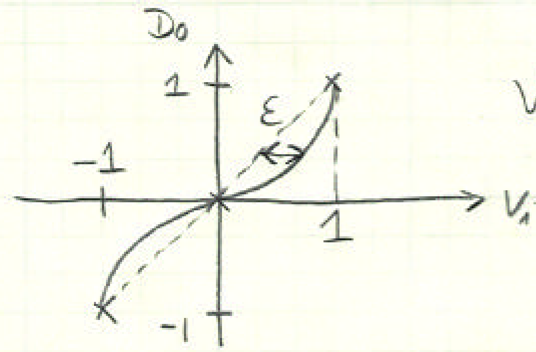
① – approximate transfer characteristics of ADC with polynomial

$$D_o = \sum_{k=0}^{\infty} a_k V_i^k$$

with:
- $a_0 = 0$ (adjusted to 0)
- $a_2 \to 0$ (no 2nd harmonic in FFT)
- ignore harmonics $>3$ (not visible)

$$\Rightarrow D_o = a_1 V_i + a_3 V_i^3 \overset{!}{=} 1 \;@\; V_i = 1$$




$$V_{FS} \overset{!}{=} 1$$

$$\Rightarrow a_1 + a_3 = 1$$

let $V_i = \sin \omega t$

$$\Rightarrow D_o = a_1 \sin \omega t + a_3 (\sin \omega t)^3$$

$$= a_1 \sin \omega t + a_3 \left( \frac{3}{4} \sin \omega t - \frac{1}{4} \sin 3\omega t \right)$$

$$= \left(a_1 + \frac{3}{4} a_3\right) \sin \omega t - \frac{1}{4} a_3 \sin 3\omega t$$

$$\Rightarrow HD_3 = \frac{\frac{1}{4} a_3}{a_1 + \frac{3}{4} a_3} = \frac{\frac{1}{4} a_3}{1 - a_3 + \frac{3}{4} a_3} = \frac{a_3}{4 - a_3}$$

$$a_3 = \frac{4\, HD_3}{1 + HD_3}$$

deviation from ideal TF:

$$V_1 - \mathcal{E} = a_1 V_1 + a_3 V_1^3$$

$$\mathcal{E} = (a_1 - 1) V_1 - a_3 V_1^3$$

$$\mathcal{E} = a_3 \left( V_1 - V_1^3 \right)$$

$$\frac{d\mathcal{E}}{dV_1} = a_3 \left( 1 - 3V_1^2 \right) = 0 \quad \Rightarrow V_1 = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \mathcal{E}_{max} = a_3 \left( \frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}} \right) = \frac{2a_3}{3\sqrt{3}}$$

$$\Rightarrow INL = \frac{\mathcal{E}_{max}}{\Delta} = \frac{\cancel{2}a_3}{3\sqrt{3}} \; \frac{2^B - 1}{\cancel{2}}$$

$$INL = \frac{1}{3\sqrt{3}} \; \frac{4\, HD_3}{1 + \cancel{HD_3}} \left( 2^B - 1 \right)$$

$$HD_3 = 10^{-42/20} = 7.94 \cdot 10^{-3}$$

$$B = \frac{SNR - 1.8}{6.02} \qquad SNR = -P_{bin} - 10 \lg 4096$$

$$= -70\,dB + 36.12\,dB$$

$$= -33.88\ dB$$

$$\Rightarrow B = \frac{33.88 - 1.8}{6.02} = 5.32 \rightarrow 5 \text{ bits ?}$$

(might actually be 5.3 bits!)

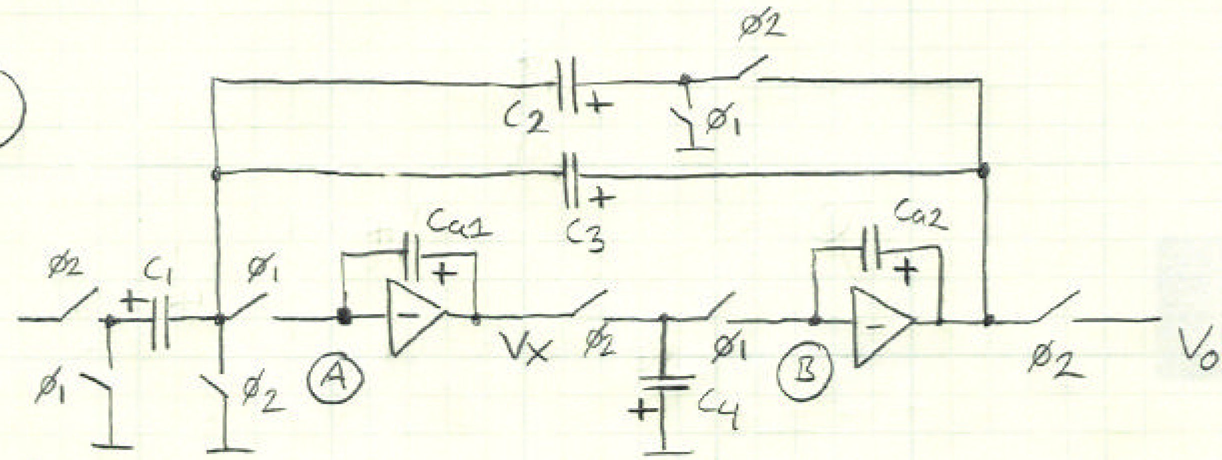
$$\Rightarrow INL \cong \frac{4}{3\sqrt{3}}\, 7.94 \cdot 10^{-3} \cdot 31 = \boxed{0.19\ LSB}$$

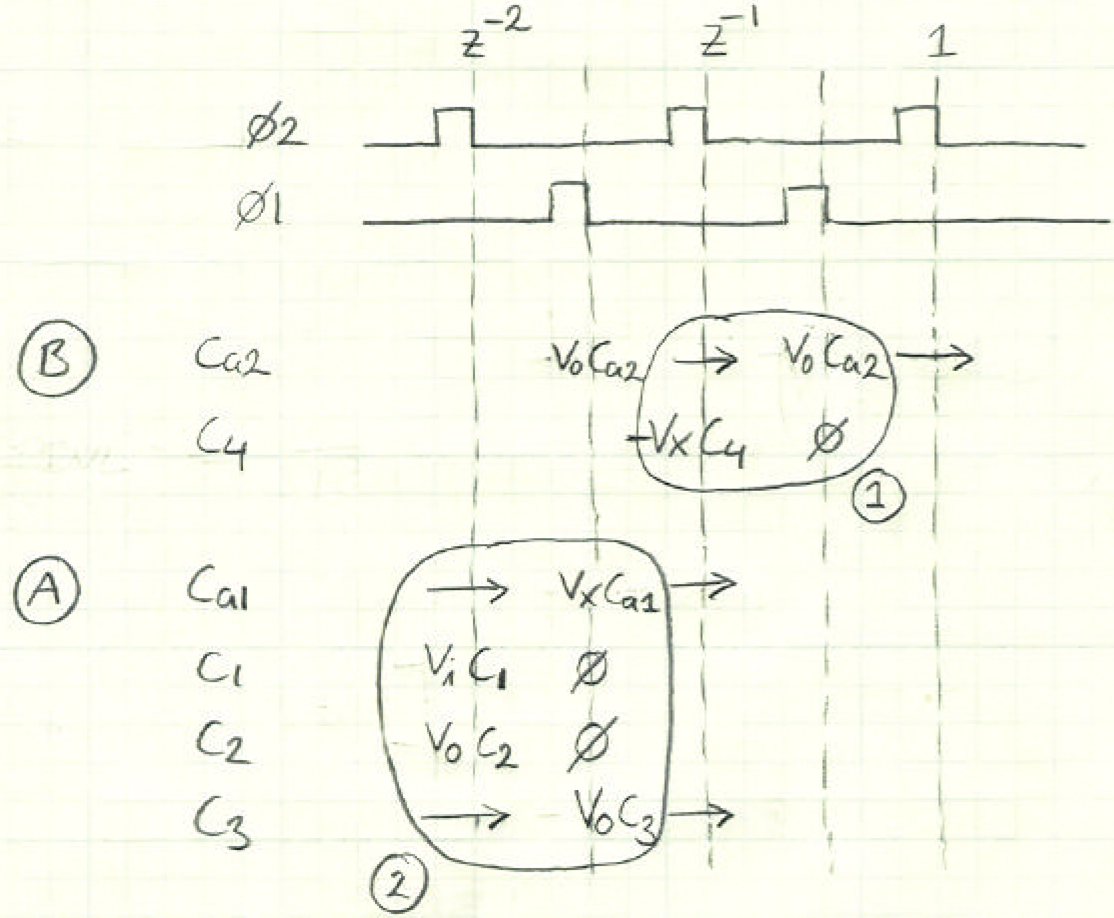
Approximate Expression: (Quick Estimate)

$$INL \cong 0.8 \cdot HD_3 \cdot 2^B = -2dB - HD_3[dB] + SNR[dB] - 1.8dB$$

$$\cong -4dB - HD_3(dB) + SNR[dB] = 0.24\ LS$$

2




$$\text{(1)} \quad V_0 C_{a2} = z^{-1} V_0 C_{a2} - z^{-1} V_x C_4$$

$$\text{(2)} \quad z^{-1} V_x C_{a1} + z^{-1} V_0 C_3 = z^{-2} V_1 C_1 + z^{-2} V_0 C_2 + z^{-2} V_0 C_3 + z^{-2} V_x C_{a2}$$

$$C_{a2} = C_{a1} \overset{!}{=} 1$$

(1)' $$z^{-1}V_x = V_0\left(\frac{1}{C_4}z^{-1} - \frac{1}{C_4}\right)$$

$$z^{-2}V_x = V_0\left(\frac{1}{C_4}z^{-2} - \frac{1}{C_4}z^{-1}\right)$$

$\Rightarrow$ (2)' $$V_0\left(\frac{1}{C_4}z^{-1} - \frac{1}{C_4}\right) + z^{-1}V_0C_3$$

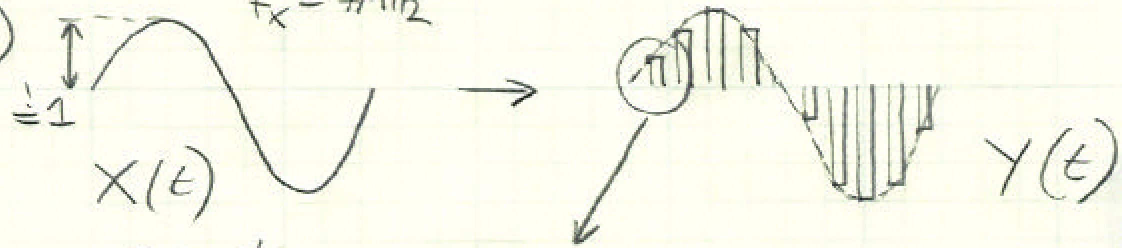
$$= z^{-2}V_1C_1 + z^{-2}V_0C_2 + z^{-2}V_0C_3 + V_0\left(\frac{1}{C_4}z^{-2} - \frac{1}{C_4}z^{-1}\right)$$

$$V_0\Big[-\frac{1}{C_4} + z^{-1}\left(C_3 + \frac{2}{C_4}\right) + z^{-2}\left(-C_2 - C_3 - \frac{1}{C_4}\right) = V_1 z^{-2}C_1$$

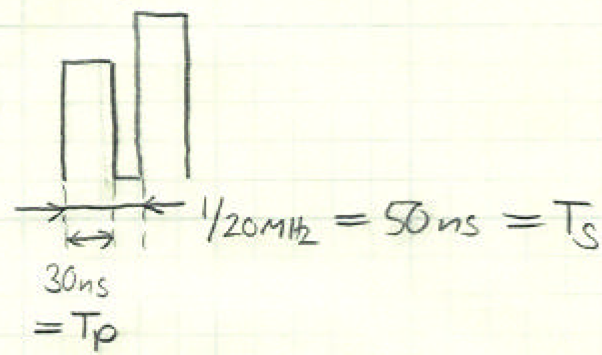
$$\frac{V_o}{V_i} = \frac{-z^{-2}C_1}{\frac{1}{C_4} - z^{-1}\left(C_3 + \frac{2}{C_4}\right) + z^{-2}\left(C_2 + C_3 + \frac{1}{C_4}\right)}$$

$$= \boxed{\frac{-z^{-2}C_1C_4}{1 - z^{-1}\left(C_3C_4 + 2\right) + z^{-2}\left[C_4\left(C_2 + C_3\right) + 1\right]}}$$

(3)

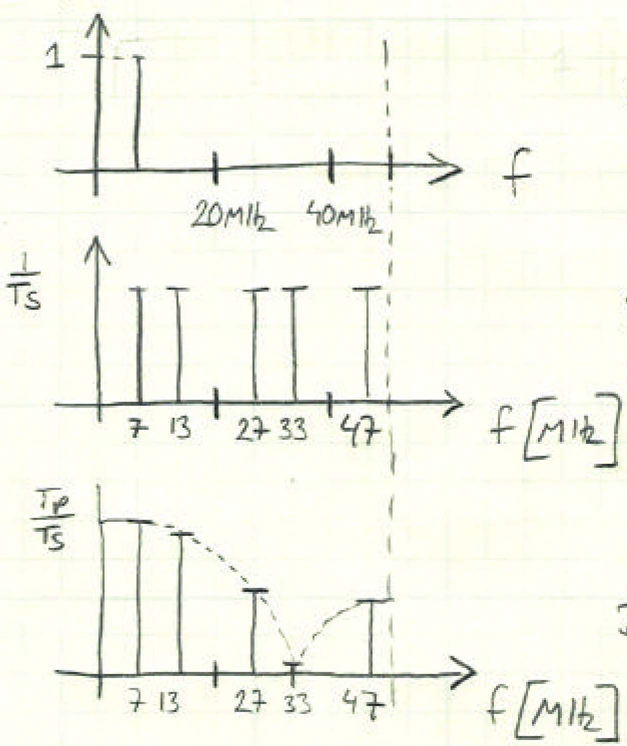



$= \cos \omega t$




$$Y(f) = T_p \frac{\sin \pi f T_p}{\pi f T_p} e^{-j\pi f T_p} \cdot \frac{1}{T_S} \sum_{n=-\infty}^{\infty} X\left(\left|f - \frac{n}{T_S}\right|\right)$$

$X(f)$




SINE

DIRAC SAMPLED SINE

$1/T_p = 33.3MHz$

DAC OUTPUT

$$\text{Amplitude}(f) = \frac{T_p}{T_s}\,\frac{\sin \pi f T_p}{\pi f T_p} \qquad \frac{T_p}{T_s} = 0.6$$

$$\text{Phase}(f) = \measuredangle\, e^{-j\pi f T_p} = -\pi f T_p$$

$$\left(\omega t + \varphi_{(\omega)}\right) \rightarrow \omega\left(t - \Delta t\right)$$

$$\Delta t = \frac{-\varphi_{(\omega)}}{\omega} = \frac{-\varphi_{(\omega)}}{2\pi f}$$

$$\text{Delay}(f) = \frac{T_p}{2} = \underline{\underline{15\,ns}} = \text{const.}$$

| Frequency [MHz] | Amplitude | Phase |
|---|---|---|
| 7 | 0.557 | $-37.8^\circ$ |
| 13 | 0.461 | $-70.2^\circ$ |
| 27 | 0.132 | $-145.8^\circ$ |
| 33 | 0.006 | $-178^\circ$ |
| 47 | 0.130 | $180^\circ - 253^\circ = -73.8^\circ$ |