EE247 Homework #6 Boris Murmann

(1) $\sigma_{INL}(max) = 2^{\frac{N}{2}-1}\,\sigma_{\Delta R/R}$, $\sigma_{\Delta R/R} = 0.2\%$

a) $N = 12$

$$\Rightarrow \sigma_{INL}(max) = 2^5 \cdot 0.2\% = 0.064\,LSB$$

$$k = \frac{0.5\,LSB}{0.064\,LSB} = 7.81$$

$\Rightarrow$ YIELD $\rightarrow$ 100 % (assuming Gaussian statistics and no gradients!)

b) $k \stackrel{!}{=} 2.6$ for 99 % yield (see e.g. G&M p. 262)

$$\Rightarrow k = \frac{0.5}{2^{\frac{N}{2}-1}\sigma_{\Delta R/R}}$$

$$\frac{N}{2} - 1 = ld\,\frac{0.5}{k\,\sigma_{\Delta R/R}}$$

$$N = 2\left(1 + ld\,\frac{0.5}{k\,\sigma_{\Delta R/R}}\right)$$

$$N = 2\left(1 + \frac{1}{\log 2}\log\frac{0.5}{k\,\sigma_{\Delta R/R}}\right)$$

$$N = 2\left(1 + \frac{1}{\log 2}\log\frac{0.5 \cdot 100}{2.6 \cdot 0.2}\right) = 15.2$$

$\Rightarrow$ Max. resolution w/o trimming $\cong$ 15 bits

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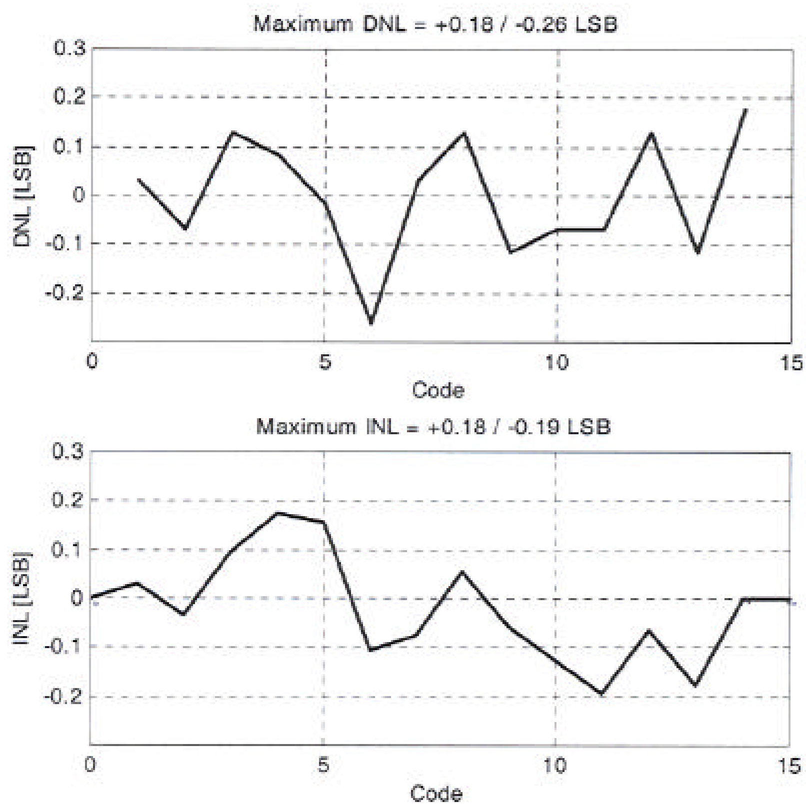
% EE247 Homework #6.2
% Boris Murmann

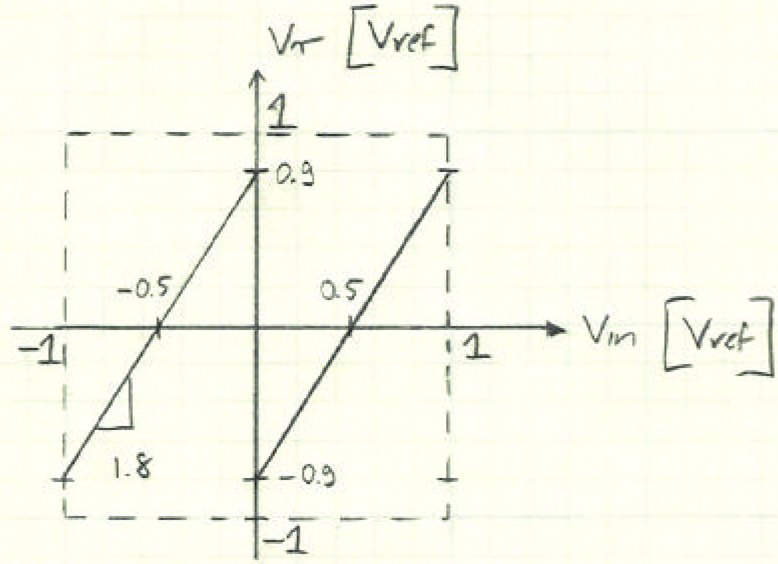
bucket= [28 21 19 23 22 20 15 21 23 18 19 19 23 18 24 29];
average = mean( bucket(2:15) )
dnl = (bucket(2:15)-average)/average;   % discard boundary codes!!
inl = zeros(1,16);
for i=2:15
   inl(i)= inl(i-1) + dnl(i-1);
end

figure(1);
subplot(2,1,1);
plot( 1:14, dnl, 'linewidth', 2 );
grid;
xlabel('Code');
ylabel('DNL [LSB]');
title(['Maximum DNL = +',num2str(max(dnl),2),' / ',num2str(min(dnl),2),' LSB
'])
axis([0 15 -0.3 0.3]);

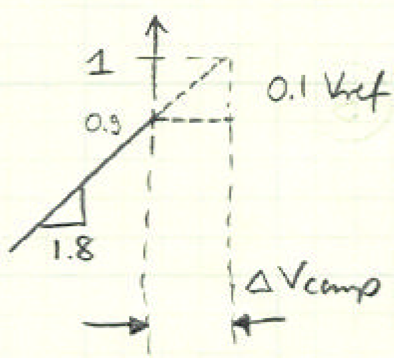
subplot(2,1,2);
plot( 0:15, inl, 'linewidth', 2 );
grid;
xlabel('Code');
ylabel('INL [LSB]');
title(['Maximum INL = +',num2str(max(inl),2),' / ',num2str(min(inl),2),' LSB
'])
axis([0 15 -0.3 0.3]);
```



③ ⓐ $$V_r = 1.8\left(V_{in} + D\,\frac{V_{ref}}{2} + \bar{D}\,\frac{V_{ref}}{2}\right)$$



ⓑ

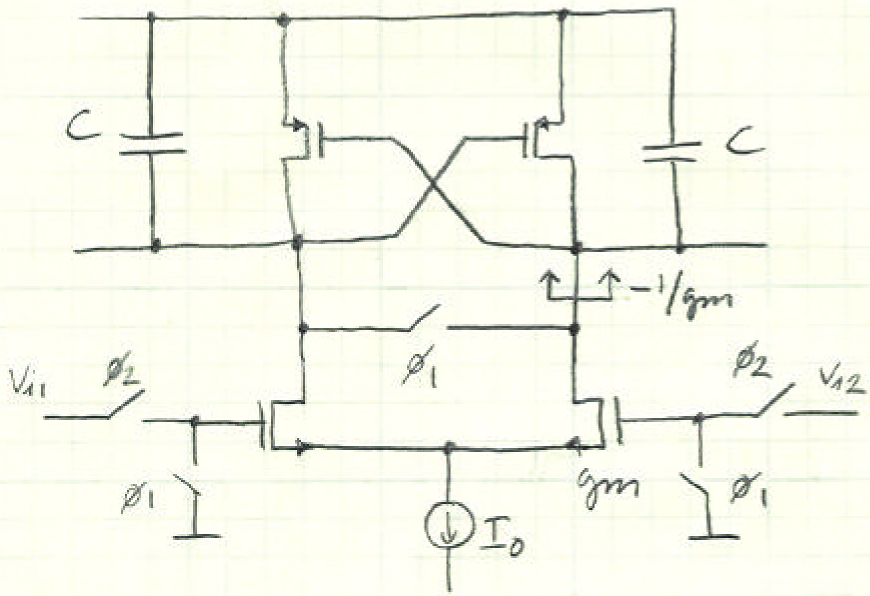


$$\Delta V_{comp} = \pm\,\frac{0.1\,V_{ref}}{1.8} = \underline{\underline{\pm\ 0.056\,V_{ref}}}$$

Vref	ΔVcomp
1V	56mV
2V	112 mV
3V	168mV

huge...!

(4)



→ diff. pair has no voltage gain

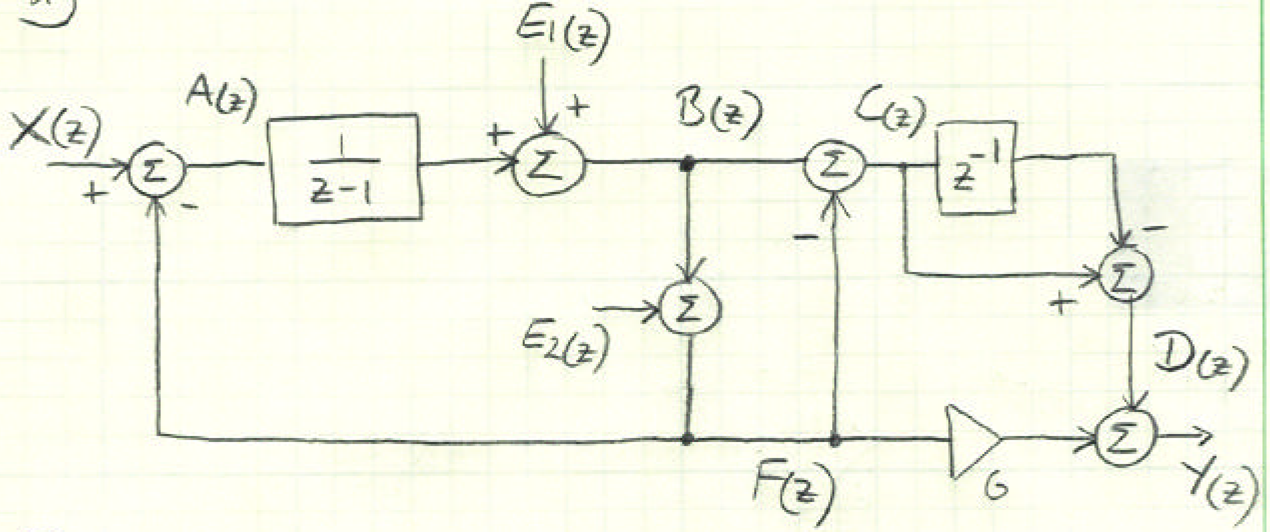
$$t_d = \frac{C}{g_m} \ln \frac{v_{od}}{v_{id}}$$

$$g_m = \frac{2 I_D}{V_{dsat}} = \frac{100\mu A}{200mV} = 0.5 mS$$

$$\Rightarrow t_d = \frac{1pF}{0.5mS} \ln \frac{1V}{1mV}$$

$$t_d = \underline{\underline{13.8\ ns}}$$

5) a)



b)

$$Y(z) = D(z) + GF(z)$$

$$D(z) = C(z)\left(1-z^{-1}\right)$$

$$= \left(B(z) - F(z)\right)\left(1-z^{-1}\right)$$

$$B(z) = \frac{A(z)}{z-1} + E_1(z)$$

$$F(z) = B(z) + E_2(z)$$

$$\Rightarrow D(z) = -E_2(z)\left(1-z^{-1}\right) //$$

$$F(z) = \frac{A(z)}{z-1} + E_1(z) + E_2(z)$$

$$A(z) = X(z) - F(z)$$

$$\Rightarrow F(z) = E_1(z) + E_2(z) + \frac{X(z)}{z-1} - \frac{F(z)}{z-1}$$

$$F(z)\left(1+\frac{1}{z-1}\right) = \frac{1}{z-1}X(z) + E_1(z) + E_2(z)$$

$$\Rightarrow F_{(z)} = \frac{z-1}{z}\left(\frac{1}{z-1}X_{(z)} + E_1(z) + E_2(z)\right)$$

$$F_{(z)} = z^{-1}X_{(z)} + \frac{z-1}{z}\left(E_{1(z)} + E_2(z)\right)$$

$$\Rightarrow Y_{(z)} = -E_2(z)\left(1-z^{-1}\right) + z^{-1}GX_{(z)} + G\,\frac{z-1}{z}\left(E_{1(z)} + E_2(z)\right)$$

$$\boxed{\begin{aligned} Y_{(z)} = z^{-1}GX_{(z)} &+ \left(1-z^{-1}\right)GE_1(z) \\ &+ \left(1-z^{-1}\right)E_2(z)\left[G-1\right] \end{aligned}}$$

$\underline{\underline{G_{opt} = +1}}$ $\Rightarrow$ removes truncation error!

c) <u>with $G = +1$:</u>

$$Y_{(z)} = z^{-1}GX_{(z)} - \left(1-z^{-1}\right)E_{1(z)}$$

NTF: $\left|\dfrac{Y_{(z)}}{E_{1(z)}}\right| = 1 - z^{-1}$

In band Q-error: $S_B = \dfrac{\pi^2}{3}\dfrac{1}{M^3} \cdot S_Q$

$$S_q = \frac{1}{12}\left(\frac{FS}{2^N-1}\right)^2$$

Signal Power: $\hat{S_x} = \dfrac{FS^2}{8}$

$$\Rightarrow D_R = \frac{\hat{S}_x}{S_B} = \frac{\cancel{FS^2}}{8} \frac{3}{\pi^2} M^3 \frac{12}{\cancel{FS^2}} \left(2^N - 1\right)^2$$

$$\boxed{DR = \frac{9}{2\pi^2} M^3 \left(2^N - 1\right)^2 \cong \frac{9}{2\pi^2} M^3 2^{2N}}$$

for $2^N \gg 1$

→ "Leslie & Singh" Architecture

→ Nonidealities usually limit $N \leq 3 \ldots 5$