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B. E. BOSER Homework 6 SOLUTION

Question 1

Download the datasheet of the AD7677 A/D converter from www.analog.com

a) Using the given code histograms for the DC input, calculate the converterís input referred thermal noise in LSB rms. (Follow the method presented in: S. Ruscak and L. Singer, ì Using Histogram Techniques to Measure A/D Converter Noise,: - available at http://www.analog.com/library/analogDialogue/archives/29-2/cnvtrnoise.html).

b) Using the data from the typical DNL plot, estimate the expected quantization noise in LSB rms error of the converter. Assume that the DNL is distributed uniformly over the bounds seen in the DNL plot.

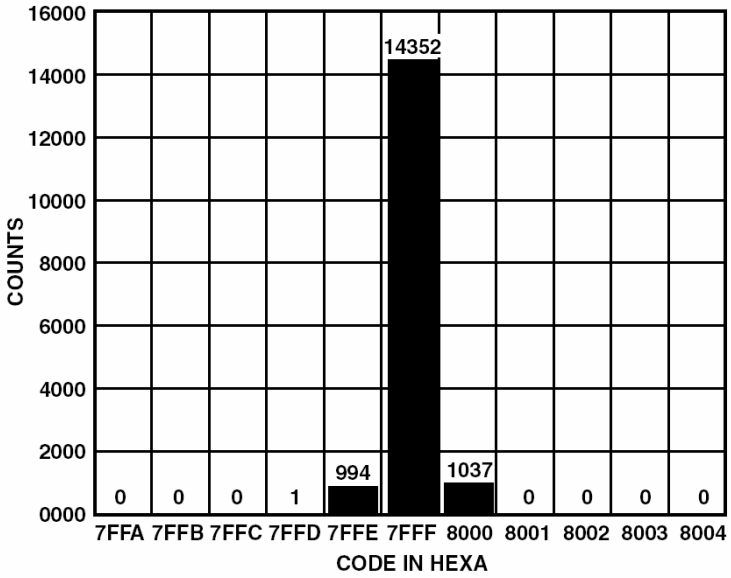
c) Combine the results in a) and b) to obtain an estimate for the converterís SNR. Does this number agree with the typical value given in the data sheet?

d) What is the ENOB of this converter?

e) The typical low frequency DFT plot of the AD7677 shows a dominant 3rd harmonic. Use your analysis from problem 1 to estimate the converterís SFDR from the peak INL seen in the given typical INL plot. Does this number agree with the specified typical SFDR value?

Solution

The code histogram is:



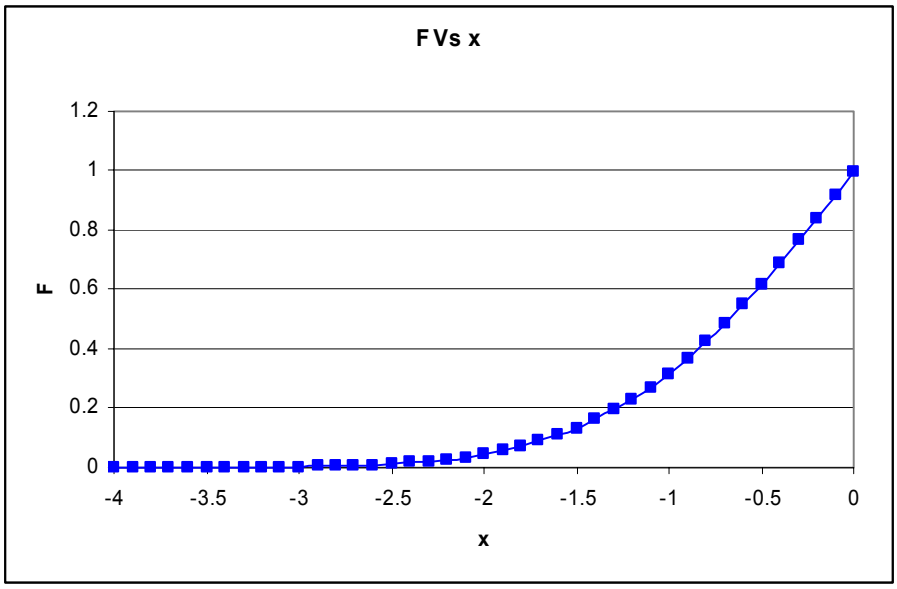
TPC 5. Histogram of 16,384 Conversions of a DC Input at the Code Center

The method used for calculating the converter noise is:

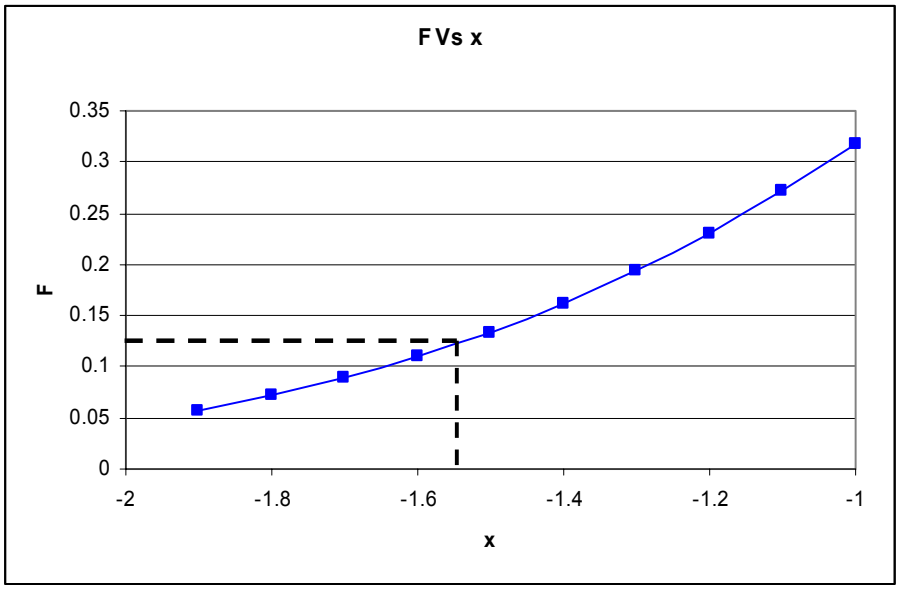
- Add the numbers of hit is each of the bins adjacent to the center bin
- Find the ratio of this number to the number of total tests
- Find x_0, the standard deviation from a stddev/Yield plot
- Find σ, from σ=-0.5/x_0 LSB

No of hits in adjacent bins = 994+1037 = 2031
Total no of tests = 994+1037+14352 = 16383
Ratio is: 2031/16383 = 0.123

Find x_0 on the Yield plots:



It is between -2 and -1



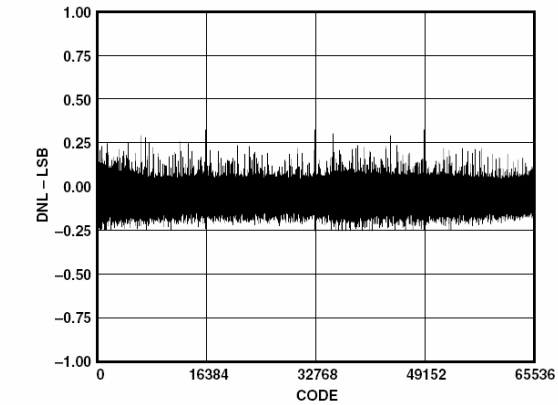
$x_0 \approx -1.55$

$$\mathbf{1.55\sigma = 0.5}\boldsymbol{LSB}$$

$$\boldsymbol{\sigma = 0.323LSB}$$

b)

From the DNL Plot, DNL=±0.25LSB



TPC 4. Differential Nonlinearity vs. Code

This is like additional quantization noise of ±0.25LSB, on top of original ±0.5LSB quantization noise. Thus, instead of $\boldsymbol{\Delta^2/12}$ we get an additional $\boldsymbol{(\Delta/2)^2/12}$.

The two noise sources add in an rms manner, thus the total rms value of the noise is =

$$\frac{\mathbf{5\cdot\Delta^2}}{\mathbf{4\cdot 12}}$$

Since N=16

$$Q_e = \sqrt{\frac{\Delta^2}{12} + \frac{(\Delta/2)^2}{12}}$$

$$Q_e = \sqrt{\frac{5}{48}}\Delta$$

$$Q_e = 0.323\Delta$$

$$Q_e = 0.323LSB$$

c)
The rms noise power is equal to the square root of the sum of the two noise contributors:

$$S_N = \sqrt{0.323^2 + 0.323^2}$$

$$S_N = \sqrt{2}(0.323)$$

$$S_N = 0.456LSB$$

The SNR is given by the peak signal power divided by the noise power:

$$SNR = \frac{\frac{1}{2}\left(\frac{2^{16}\Delta}{2}\right)^2}{(0.456\Delta)^2}$$

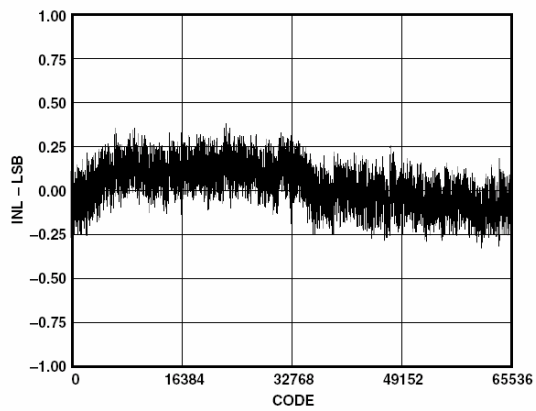
$$SNR = 2.582E9$$

SNR=94.1dB

Yes! This does agree with the SNR given in the datasheet.

d)ENOB=(SNR-1.76)/6.02 = 15.3 bits

e)
Using the inverse of the technique used in HW4 Q2:
From the typical INL plot we see that INL = 0.25LSB



TPC 1. Integral Nonlinearity vs. Code

➔ Verror = $0.25V_{FS}/2^{15}$

➔ $=V_{FS}/2^{17}$

$v_{error}=2a_3(1/\text{sqrt}(3))^3$

➔ $a_3 = \text{verror}*\text{sqrt}(3)^3/2$

➔ $=19.8E–6$

$SFDR=(a_3/4)/(1+3a_3/4)$

$\cong a_3/4$

$=19.8E\text{-}6/4$

$=4.955E\text{-}6$

$=20\log_{10}(4.955E\text{-}6) = -106dB$

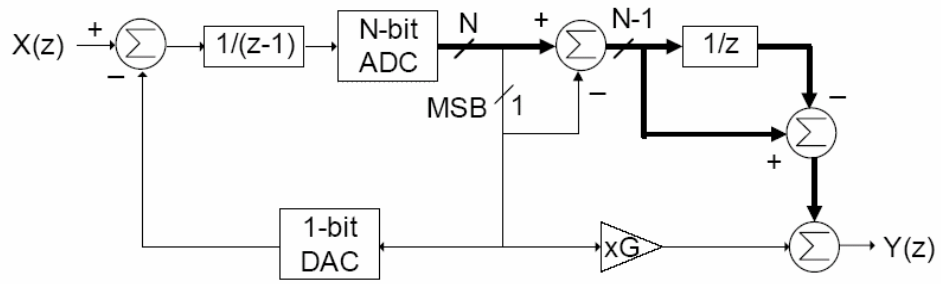
This is close to the -109dB figure in the datasheet.

Question 2

The sigma-delta modulator below employs an N-bit ADC for increased resolution, but only a 1-bit DAC is used toavoid distortion due to DAC nonlinearity. The remaining N-1 bits of ADC outputs serve as an estimate of the quantization error.

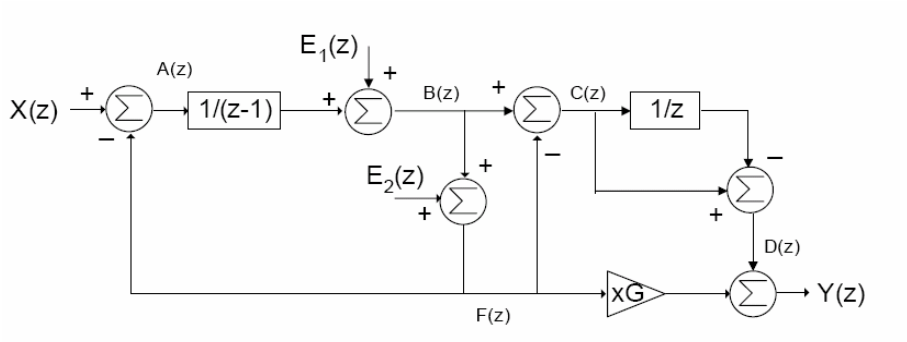
a) Find Y(z) as a function of the input X(z), quantization error $E_1(z)$ and the ADC truncation error $E_2(z)$. What is the optimal value for G in order to eliminate the truncation error in Y(z)?

b) Compute the dynamic range of the converter as a function of N and the oversampling ratio M. Assume optimal setting of G calculated in a) and an ideal (i.e. brickwall) decimation filter.



Solution:

Consider the simplified block diagram below:



In particular, the transfer function of X(z), $E_1(z)$ and $E_2(z)$ to F(z) are examined.

The transfer function for X(z) to F(z) in general is $\dfrac{GH(z)}{1+GH(z)}$.

In this case G=1, $H(z)=\dfrac{1}{z-1}$

Therefore,

$$\frac{F(z)}{X(z)}=\frac{\frac{1}{z-1}}{1+\frac{1}{z-1}}=z^{-1}$$

The transfer function for both $E_1(z)$ and $E_2(z)$ to F(z) is $\dfrac{G}{1+GH(z)}$, where, as above, in this case G=1, $H(z)=\dfrac{1}{z-1}$.

Thus,

$$\frac{F(z)}{E_1(z)}=\frac{1}{1+\frac{1}{z-1}}=1-z^{-1}$$

Therefore,

$$F(z)=z^{-1}X(z)+(1-z^{-1})E_1(z)+(1-z^{-1})E_2(z)$$

Since,

$$F(z)=B(z)+E_2(z)$$

This implies,

$$B(z)=F(z)-E_2(z)$$

$$B(z)=z^{-1}X(z)+(1-z^{-1})E_1(z)-z^{-1}E_2(z)$$

$$C(z)=B(z)-F(z)$$

$$C(z)=-E_2(z)$$

$$D(z) = (1 - 1/z)C(z)$$
$$D(z) = (z^{-1} - 1)E_2(z)$$

$$Y(z) = D(z) + G \cdot F(z)$$
$$Y(z) = (z^{-1} - 1)E_2(z) + G(z^{-1}X(z) + (1 - z^{-1})E_1(z) + (1 - z^{-1})E_2(z))$$
$$Y(z) = G(z^{-1}X(z) + G(1 - z^{-1})E_1(z)) + (1 - z^{-1})(G - 1)E_2(z)$$

In order to eliminate the $E_2(z)$ term, this implies G=1.

b)
G=1 $\Rightarrow$
$$Y(z) = z^{-1}X(z) + (1 - z^{-1})E_1(z)$$

$$NTF(z) = 1 - z^{-1}$$
$$|NTF(z)|^2 = NTF(z)NTF(z^{-1})$$
$$|NTF(z)|^2 = (1 - z^{-1})(1 - z)$$
$$|NTF(z)|^2 = 1 - z^{-1} - z + 1$$
$$|NTF(z)|^2 = 2 - 2\cos\omega T$$
$$|NTF(z)|^2 = (2\sin\pi fT)^2$$

Noise power, $\overline{S_Y}$, is given by:
$$\overline{S_Y} = \int_{-B}^{B} S_Q(f)|NTF(z)|^2 df$$
Where, $S_Q(f)$ is the noise power spectral density, and B is the desired bandwidth.
In this case,
$$S_Q(f) = \frac{1}{f_s} \cdot \frac{\Delta^2}{12}$$
$$B = \frac{f_s}{M}$$
Therefore,
$$\overline{S_Y} = \int_{-f_s/M}^{f_s/M} \frac{1}{f_s} \cdot \frac{\Delta^2}{12} (2\sin\pi fT)^2 df$$
if M>>1, then f<<1/T, and $\sin\pi fT \approx \pi fT$
$$\overline{S_Y} = \frac{\pi^2}{3} \frac{1}{M^3} \frac{\Delta^2}{12}$$
The peak signal power is given by:

$$\overline{S_X} = \frac{1}{2}\left(\frac{2^N \Delta}{2}\right)^2$$

The Dynamic Range, DR, is:

$$DR = \frac{\overline{S_X}}{\overline{S_Y}}$$

$$DR = \frac{9}{\pi^2} M^3 2^{2N-1}$$