2^{nd} Order Transfer Functions

- Imaginary axis zeroes
- Tow-Thomas Biquad
- Example

Imaginary Axis Zeros

- Sharpen transition band
- "notch out" interference
- High-pass filter (HPF)
- Band-reject filter

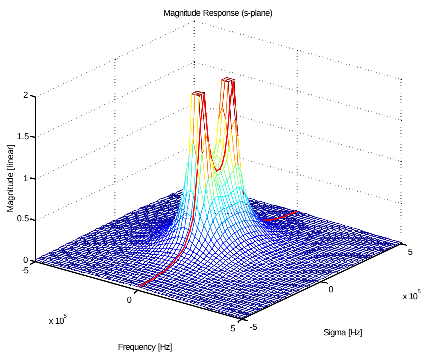
$$H(s) = K \frac{1 + \left(\frac{s}{w_Z}\right)^2}{1 + \frac{s}{w_P Q_P} + \left(\frac{s}{w_P}\right)^2}$$

$$\left|H(jw)\right|_{w \to \infty} = K \left(\frac{w_P}{w_Z}\right)^2$$

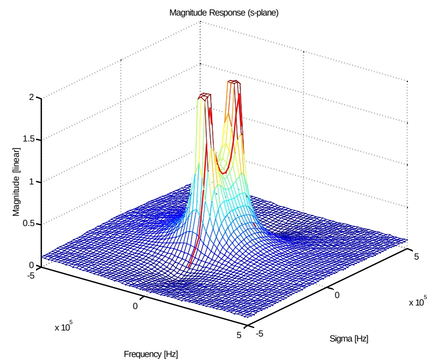
Note: Always represent transfer functions as a product of a gain term, poles, and zeros (pairs if complex). Then all coefficients have a physical meaning, reasonable magnitude, and easily checkable unit.

Imaginary Axis Zeros

No finite zeros



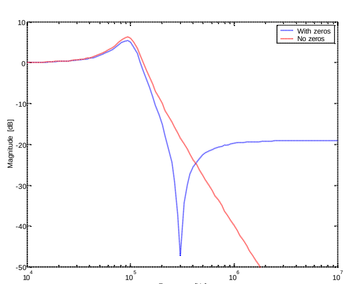
With finite zeros

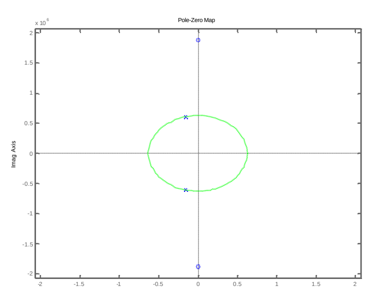


Imaginary Zeros

$$f_P = 100kHz$$
$$Q_P = 2$$
$$f_Z = 3f_P$$

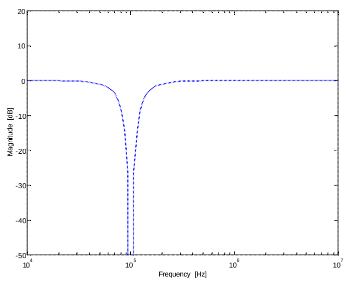
- Zeros substantially sharpen transition band
- At the expense of reduced stop-band attenuation at high frequency

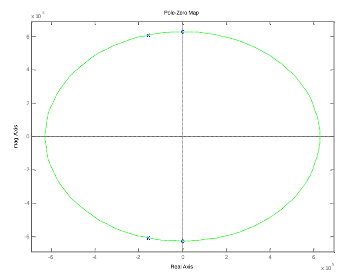




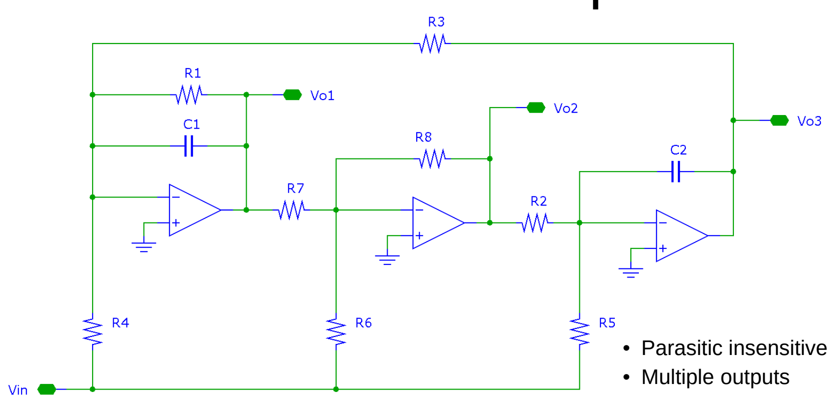
Moving the Zeros

$$f_P = 100kHz$$
$$Q_P = 2$$
$$\underline{f_Z = f_P}$$





Tow-Thomas Biquad



- Parasitic insensitive
- Multiple outputs

Ref: P. E. Fleischer and J. Tow, "Design Formulas for biquad active filters using three operational amplifiers," Proc. IEEE, vol. 61, pp. 662-3, May 1973.

Frequency Response

$$\frac{V_{o1}}{V_{in}} = -k_2 \frac{(b_2 a_1 - b_1)s + (b_2 a_0 - b_0)}{s^2 + a_1 s + a_0}$$

$$\frac{V_{o2}}{V_{in}} = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$$

$$\frac{V_{o3}}{V_{in}} = -\frac{1}{k_1 \sqrt{a_0}} \frac{(b_0 - b_2 a_0)s + (a_1 b_0 - a_0 b_1)}{s^2 + a_1 s + a_0}$$

- V_{o2} implements a general biquad section with arbitrary poles and zeros
- V_{o1} and V_{o3} realize the same poles but are limited to at most one finite zero

Component Values

$$b_0 = \frac{R_8}{R_3 R_5 R_7 C_1 C_2}$$

$$b_1 = \frac{1}{R_1 C_1}\left(\frac{R_8}{R_6} - \frac{R_1 R_8}{R_4 R_7}\right)$$

$$b_2 = \frac{R_8}{R_6}$$

$$a_0 = \frac{R_8}{R_2 R_3 R_7 C_1 C_2}$$

$$a_1 = \frac{1}{R_1 C_1}$$

$$k_1 = \sqrt{\frac{R_2 R_8 C_2}{R_3 R_7 C_1}}$$

$$k_2 = \frac{R_7}{R_8}$$

given a_i, b_i, k_i, C_1, C_2 and R_8

$$R_1 = \frac{1}{a_1 C_1}$$

$$R_2 = \frac{k_1}{\sqrt{a_0} C_2}$$

$$R_3 = \frac{1}{k_1 k_2} \frac{1}{\sqrt{a_0} C_1}$$

$$R_4 = \frac{1}{k_2} \frac{1}{a_1 b_2 - b_1} \frac{1}{C_1}$$

$$R_5 = \frac{k_1 \sqrt{a_0}}{b_0 C_2}$$

$$R_6 = \frac{R_8}{b_2}$$

$$R_7 = k_2 R_8$$

it follows that

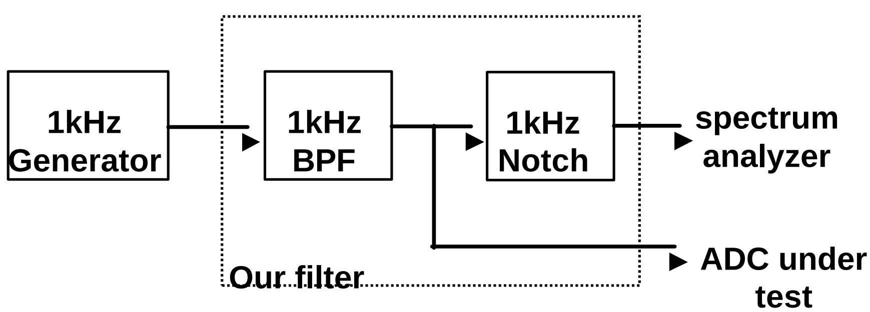
$$w_P = \sqrt{\frac{R_8}{R_2 R_3 R_7 C_1 C_2}}$$

$$Q_P = w_P R_1 C_1$$

Filter Design Example

- Application: testing of ultra-linear ADC
- Problem: sinusoidal source has higher distortion than the ADC!
- Solution
  - Filter source with bandpass before converting
  - Check resulting source with spectral analyzer
    Twist: the analyzer is not sufficiently linear either
    → notch out sinusoid and look just at harmonics
- Implementation
  - Bandpass & Notch at 1kHz
  - Use V_{o2} for bandpass (only possibility), V_{o1} for notch

Filter Design Example



Principle: IC test circuits are useless if you can't verify their performance!

Filter Coefficients

$$\frac{V_{o1}}{V_{in}} = -k_2 \frac{(b_2 a_1 - b_1)s + (b_2 a_0 - b_0)}{s^2 + a_1 s + a_0}$$

$$\frac{V_{o2}}{V_{in}} = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$$

$$\frac{V_{o3}}{V_{in}} = -\frac{1}{k_1 \sqrt{a_0}} \frac{(b_0 - b_2 a_0)s + (a_1 b_0 - a_0 b_1)}{s^2 + a_1 s + a_0}$$

Design Notch:

$$b_0 = a_0 = w_P^2 = (2p \times 1kHz)^2$$
$$b_1 = 0$$
$$b_2 = 1$$

Get Bandpass for "free":

$$b_2 a_1 - b_1 = a_1$$
$$b_2 a_0 - b_0 = 0 \quad \text{(just as we want in a bandpass)}$$

Choose reasonable signal levels:

$k_1 = 1.05$ (to keep unused V_{o3} slightly below other outputs)

$k_2 = 1$

Final Filter

$$\frac{V_{o1}}{V_{in}} = -\frac{a_1 s}{s^2 + a_1 s + a_0}$$

$$\frac{V_{o2}}{V_{in}} = \frac{s^2 + a_0}{s^2 + a_1 s + a_0}$$

$$\frac{V_{o3}}{V_{in}} = -\frac{1}{1.05} \frac{a_1 \sqrt{a_0}}{s^2 + a_1 s + a_0}$$

Choose:

C1=C2=112nF (large to minimize noise)
R8=1kΩ
f_P=1kHz, Q_P=30 (check sensitivity!)

Solve equations …

R1=42.631kΩ
R2=1.4921kΩ
R3=1.3534kΩ
R4=42.631kΩ
R5=1.4921kΩ
R6=R7=R8

Let's order the parts …

Capacitors

- C0G capacitors
  - Vishay Vitramon, C0G Dielectric Capacitor datasheet, 2000. http://www.vishay.com/document/45002/45002.pdf
  - Negligible voltage coefficient (for linearity)
  - Excellent tempco (30ppm/°C)
  - 2% initial accuracy is easy to get
- No high-value capacitors are trimmable
- Resistors will be trimmed to compensate for capacitor variations

Resistors

- Trimmed resistors combine fixed metal film resistors and precision trim potentiometers in series
  - 1%-accurate, 5ppm/°C, lab grade metal film resistors provide ~90% of the nominal resistance

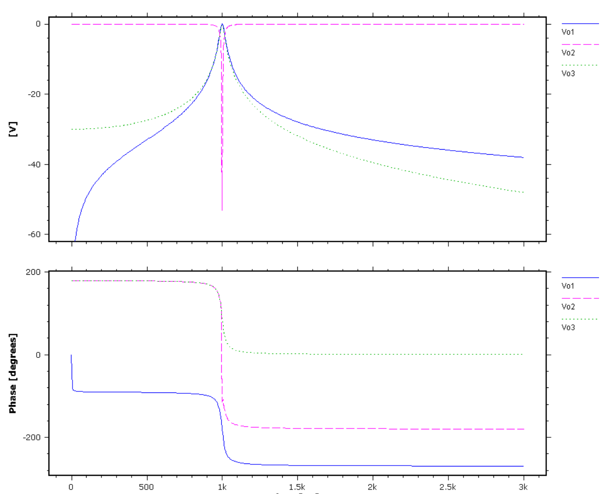
    Ref: Caddock Electronics, Type TN Lab Grade Low TC Precision Film Resistor datasheet, 1999.
  - 50ppm/°C trim pots provide between 0% and ~20% of the nominal resistance

    Ref: Vishay Foil Resistors, Model 1268 Precision Trimming Potentiometers datasheet
  - Use two fixed resistors in series with the trimpot to minimize trimpot value and optimize overall tempco
- R6-R8 are 0.1%-accurate, 5ppm/°C metal film

Opamps

- For opamps, we'll use the Burr-Brown OPA627
  - Ref: Texas Instruments / Burr-Brown, OPA627 and OPA604 datasheets, 1989.
  - The finest audio opamp in the world, and, at $15/each, priced accordingly!
  - But money is no object when designing IC test fixtures (only a few are ever built)
  - Adequate speed for this application

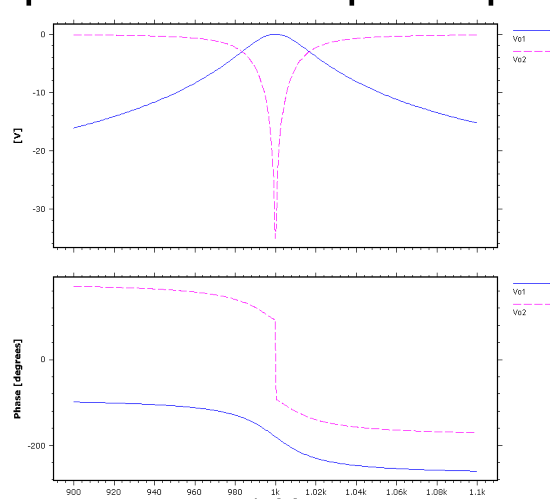
Bandpass/Bandstop Responses



Filter Design Example (cont.)

- Note that the bandpass output H_1 provides >30dB attenuation to all harmonics present in the 1kHz generator output
- Opamp outputs have 0.0±0.5dB peak gain
  - This maximizes each opamp's output swing for best dynamic range
- Let's magnify the frequency axis for the two responses of interest…

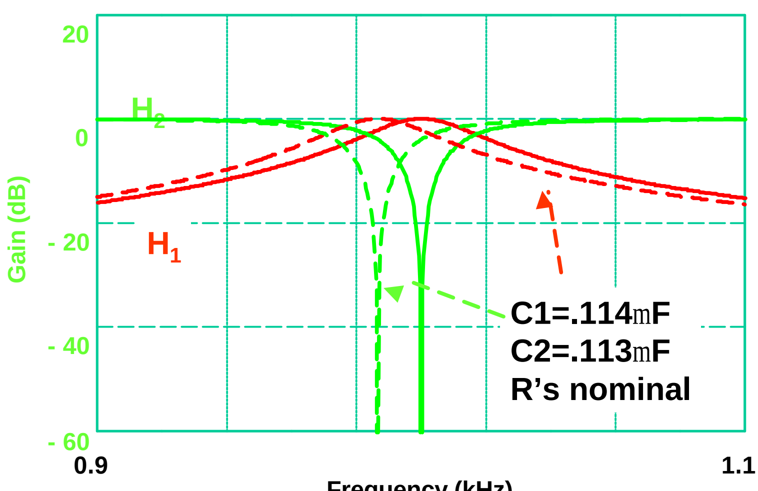
Bandpass/Bandstop Responses



Filter Design Example

- Temperature changes won't change these responses too much
  - Lab temperatures are stable to 25±3°C
  - Our lab-grade RC products move <100ppm/°C
- Initial component values are another story
  - What if C1=114nF and C2=113nF?
  - That's within their ±2% accuracy specifications
  - What's $S_{C_1}^{w_P}$?

Bandpass/Bandstop Responses



Filter Design Example

- Obviously, we've got to tune the filter back to its original specification
- How is that tuning done?
  – Do you tell your technician to twiddle pots randomly until it works?
  – Or do you document a robust tuning procedure?

RC Filter Tuning Strategy

- Famous biquads like the Tow-Thomas come complete with their own tuning strategies
  – The circuit topologies allow 1 trim operation to adjust 1 design parameter (such as f_P, f_Z, Q_P, Q_Z, gain) without changing the others
- Rationale for a biquad's tuning strategy becomes apparent when studying design equations such as the Tow-Thomas equations on slide 6

Tow-Thomas Tuning Strategy

- R3 will be set to a fixed value to keep the unused OPAMP3 output below 0dB
- Tuning involves the following steps performed in the specified sequence:
  - Adjust R2 to center the bandpass at 1kHz
  - Adjust R5 to center the notch at 1kHz
  - Adjust R1 to set the bandpass Q to 30
  - Adjust R4 to deepen the notch

Tow-Thomas Tuning Strategy

- The design equations also provide the range of adjustment required for a given resistor
  - Remember that an excessively large adjustment range translates into excessively large tempco
- R1 tuning range (from slide 7):

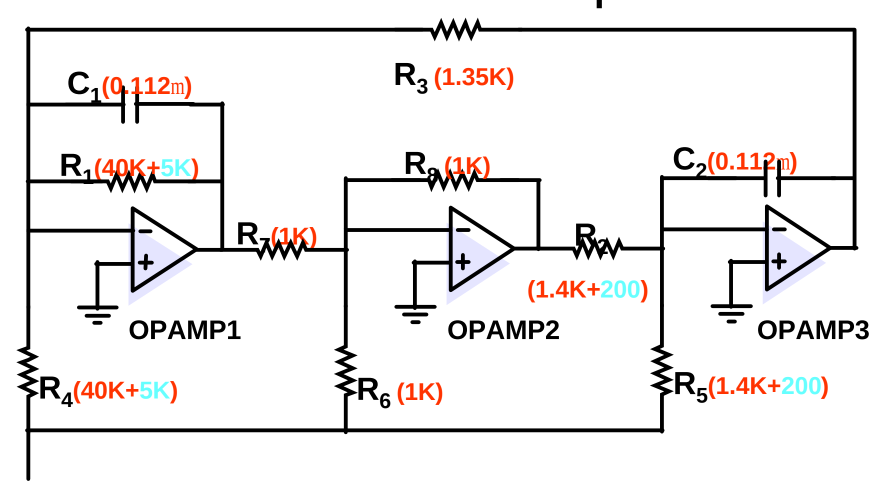
$$a_1 ° \frac{1}{R_1 C_1} \quad þ \quad \frac{1}{a_1 C_{1MAX}} < R_1 < \frac{1}{a_1 C_{1MIN}}$$

known — set by capacitor tolerances

Tow-Thomas Tuning Strategy

- An even simpler way to determine resistor ranges is to:
  – Set all capacitors to their high tolerance limit (nominal+2% in this case)
  – Calculate R's for these capacitances (these will be the minimum resistance values)
  – Set capacitors to their low tolerance limit
  – Calculate maximum R's

Tow-Thomas Biquad

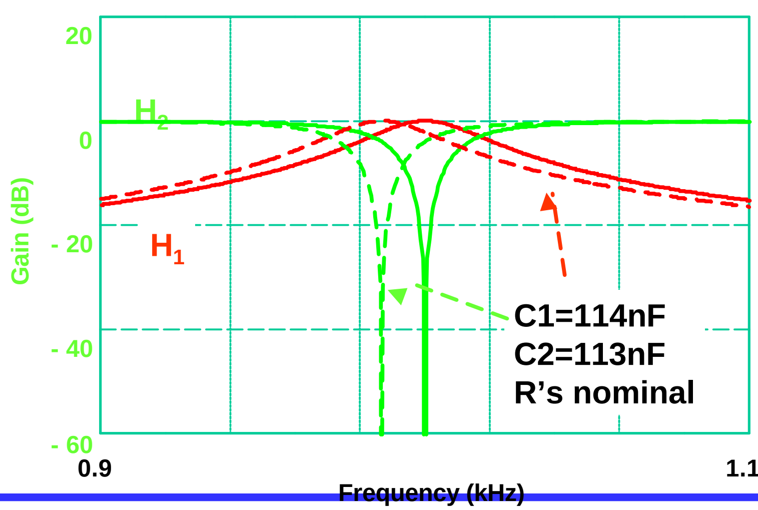


resistors: metal film + trimpot

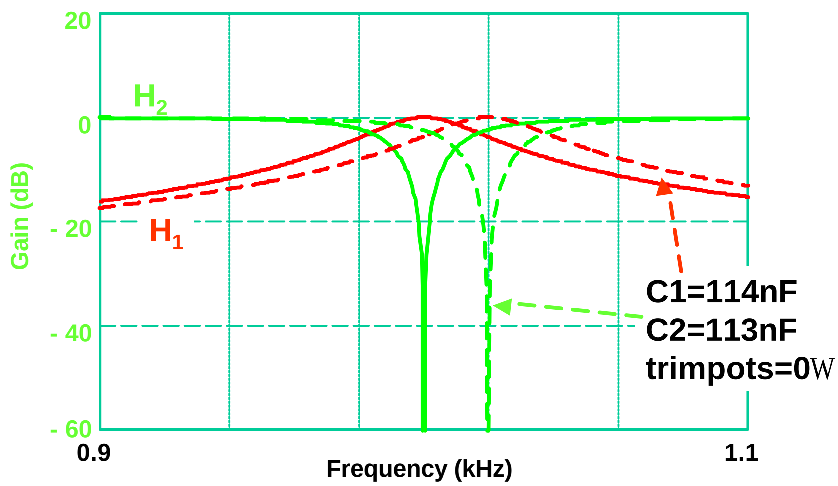
Tow-Thomas Tuning Strategy

- If you've left your filter unattended for a while, assume that its trim potentiometers are completely misadjusted
- Adjust all trimpots to 0Ω and start over
  - Let's return to our C1=114nF, C2=113nF example

Bandpass/Bandstop Responses



Bandpass/Bandstop Responses



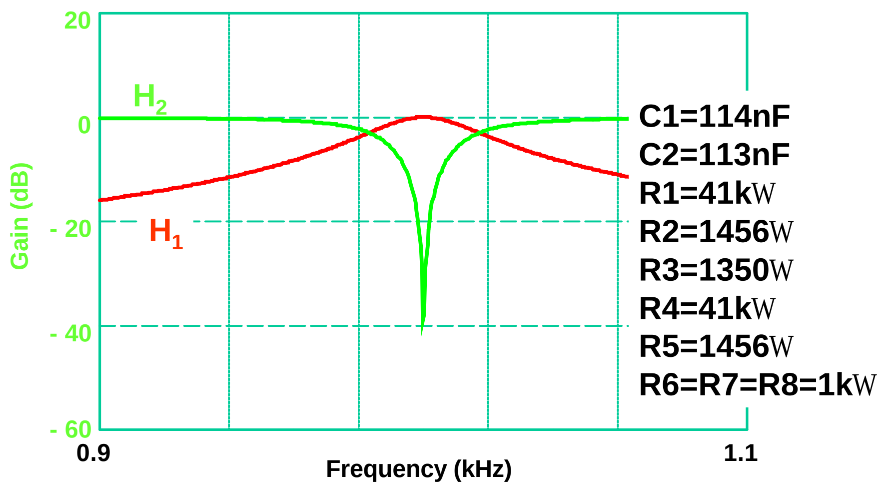
Tow-Thomas Tuning Strategy

- For most R's and C's in this biquad $f_P \sim 1/\sqrt{x}$
- Hence,

$$S_x^{f_P} = -\frac{1}{2}$$

- This means a +2% change in R_2 will cause a –1% change in f_P
- Note that f_Z sensitivities are also –1/2
  - A 4% increase in R_5 will shift our notch (currently at 1.02kHz) back to the right place

Bandpass/Bandstop Responses



Summary

- General 2nd order transfer function
  - Imaginary axis zeros
- General purpose biquad
  - Large selection in literature
  - Tow-Thomas biquad:
    - 3 opamps
    - Parasitic insensitive
    - Multiple outputs
    - Tuning strategy