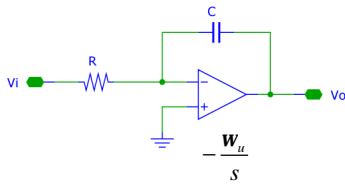


Implementation Issues

- Component spread
- Sensitivity
- Tuning
- Noise
- Finite Gain
- Finite Bandwidth
- ...

Finite Bandwidth



$$\frac{V_o}{V_i} = \underbrace{-\frac{w_o}{s}}_{\text{ideal}} \underbrace{\frac{w_u}{w_o + w_u}}_{\text{DC gain error}} \underbrace{\frac{1}{1 - \frac{s}{w_o + w_u}}}_{\text{pole}}$$

with $w_o = \frac{1}{RC}$

- Model finite bandwidth only
- DC gain infinite (keep things manageable & stand a chance to actually learn something from the calculation)
- Errors:
 - DC Gain: minor problem
 - Pole: → phase shift
- What's the effect on a filter?

Effect on Filter Response

Ideal Integrator $H_{\text{int_ideal}} = -\frac{w_o}{s}$

Actual Integrator $H_{\text{int_actual}} \cong -\frac{w_o}{s} \frac{1}{1 + \frac{s}{p_2}}$ with $p_2 = -w_u - w_o \cong -w_u$

Ideal Filter $H_{\text{filter_ideal}}(s) \Big|_{s=p_{\text{ideal}}} \rightarrow \infty$

Actual Filter $H_{\text{filter_actual}} \left(s \left(1 + \frac{s}{p_2} \right) \right) \Big|_{s=p_{\text{actual}}} \rightarrow \infty$

Hence: $p_{\text{ideal}} = p_{\text{actual}} \left(1 + \frac{p_{\text{actual}}}{p_2} \right)$
Solve for p_{actual} !

Solving for p_{actual}

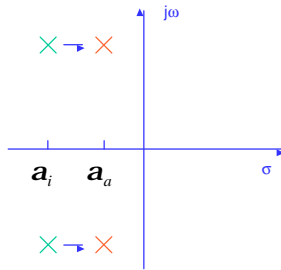
$$\begin{aligned}
 p_{\text{ideal}} &= \mathbf{a}_i + j\mathbf{b}_i \\
 p_{\text{actual}} &= \mathbf{a}_a + j\mathbf{b}_a \\
 p_2 &= -\mathbf{a}_2
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{a}_i + j\mathbf{b}_i &= (\mathbf{a}_a + j\mathbf{b}_a) \left(1 + \frac{\mathbf{a}_a + j\mathbf{b}_a}{\mathbf{a}_2} \right) \\
 \mathbf{a}_i &= \mathbf{a}_a \left(1 + \frac{\mathbf{a}_a}{\mathbf{a}_2} \right) - \frac{\mathbf{b}_a^2}{\mathbf{a}_2} \\
 &\cong \mathbf{a}_a - \frac{\mathbf{b}_a^2}{\mathbf{a}_2} \\
 \mathbf{a}_a &\cong \mathbf{a}_i + \frac{\mathbf{b}_a^2}{\mathbf{a}_2} = \mathbf{a}_i \left(1 + \frac{\mathbf{b}_a \mathbf{b}_a}{\mathbf{a}_i \mathbf{a}_2} \right) \\
 &\cong \mathbf{a}_i \left(1 - 2Q_i \frac{\mathbf{b}_a \mathbf{b}_a}{\mathbf{a}_i \mathbf{a}_2} \right) \quad \text{using } \frac{\mathbf{b}}{\mathbf{a}} = -2Q \\
 &\cong \mathbf{a}_i \left(1 - 2Q_i \frac{\mathbf{b}_a}{\mathbf{a}_2} \right) \quad \text{assuming } \mathbf{b}_a \approx \mathbf{b}_i \\
 &\cong \mathbf{a}_i \left(1 - 2Q_i \frac{w_{\text{corner}}}{\mathbf{a}_2} \right)
 \end{aligned}$$

$|b_i| \gg |a_i|$ (high Q)
 $|b_a| \gg |a_a|$
 $|a_2| \gg |a_i|, |a_a|$

Effect of Finite Amplifier Bandwidth

$$a_a \cong a_i \left(1 - 2Q_i \frac{w_{\text{corner}}}{a_2} \right)$$



Example :

$$Q = 10, f_{\text{corner}} = 1\text{MHz}, f_u = 100\text{MHz}$$

$$a_a \cong a_i \left(1 - 2 \times 10 \frac{1\text{MHz}}{100\text{MHz}} \right) = 0.8a_i$$

To reduce $a < 1\%$ (and, hence increase Q by $< 1\%$) need

$$f_u > 200Qf_{\text{corner}}$$

Solution:

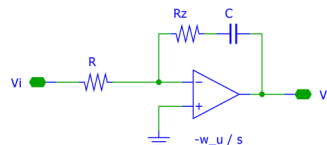
- Very fast amplifier, or
- Compensation circuit

Q enhancement \rightarrow peaking in response (or instability)



Pole-Zero Cancellation

$$\begin{aligned}
 H &= - \frac{1 + sR_z C}{sRC \left(1 + \frac{s}{w_u} \right) + \frac{s}{w_u} (1 + sR_z C)} \\
 &= \frac{1}{\frac{sRC}{\text{ideal response}}} \underbrace{\frac{1 + sR_z C}{1 + \frac{s}{w_u} + \frac{1}{w_u RC} (1 + sR_z C)}}_{\text{error, should be 1}} \\
 &= - \frac{w_o}{s} \frac{1 + sR_z C}{1 + \frac{w_o}{w_u} + \frac{s}{w_u} \left(1 + \frac{R_z}{R} \right)} \\
 &= - \frac{w_o}{s} \frac{1}{1 + \frac{w_o}{w_u}} \frac{1 + sR_z C}{1 + s \frac{\left(1 + \frac{R_z}{R} \right)}{1 + \frac{w_o}{w_u}}} \\
 &= - \frac{w_o}{s} \frac{1}{1 + \frac{w_o}{w_u}} \frac{1 + sR_z C}{w_u \left(1 + \frac{w_o}{w_u} \right)}
 \end{aligned}$$



$$\begin{aligned}
 \text{Need } & \frac{\left(1 + \frac{R_z}{R} \right)}{w_u \left(1 + \frac{w_o}{w_u} \right)} = R_z C \\
 \Rightarrow R_z & \cong \frac{1}{w_u C} \quad \text{assuming } w_u \gg w_o
 \end{aligned}$$

