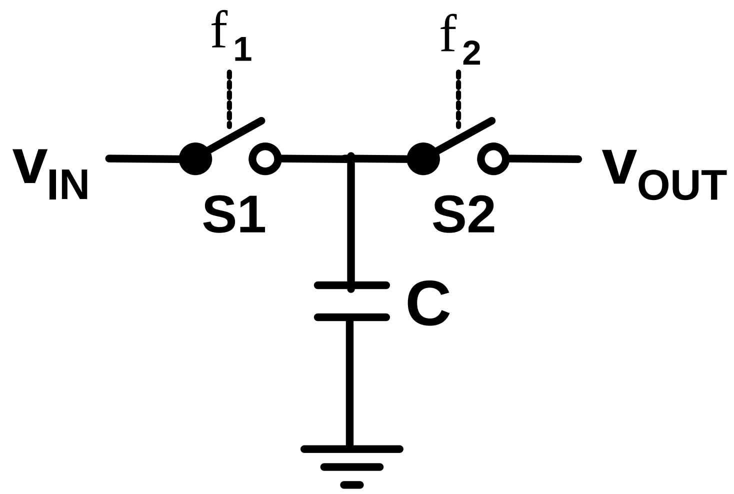
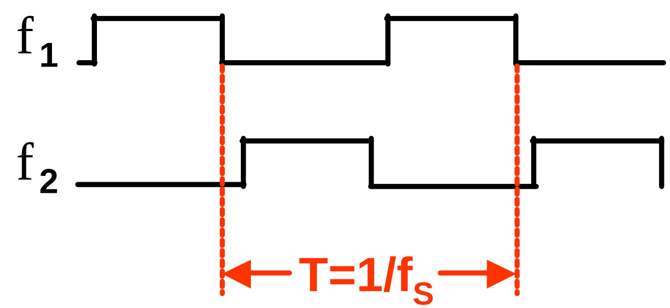
Switched-Capacitor Filters

- "Analog" sampled-data filters:
  - Continuous amplitude
  - Quantized time
- Applications:
  - Oversampled A/D and D/A converters
  - Analog front-ends (CDS, etc)
  - Standalone filters
    E.g. National Semiconductor LMF100
  - Replaced by ADC + DSP in many cases

Switched-Capacitor Resistor

- Capacitor C is the "switched capacitor"
- Non-overlapping clocks ϕ_1 and ϕ_2 control switches S1 and S2, respectively
- v_{IN} is sampled at the falling edge of ϕ_1
  - Sampling frequency f_S
- Why is this a resistor?

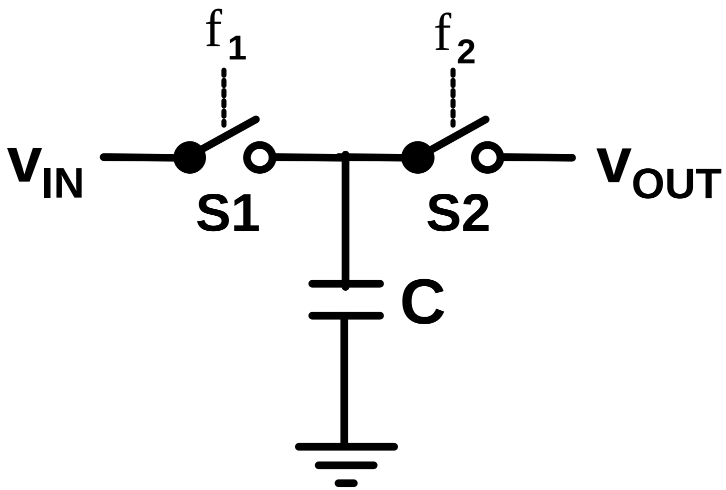




Switched-Capacitor Resistors

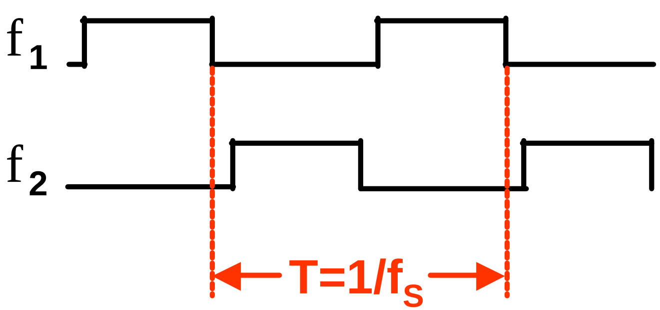
- The charge transferred from v_{IN} to v_{OUT} each sample period is:

  $$Q = C(v_{IN} - v_{OUT})$$



- The average current flowing from v_{IN} to v_{OUT} is:

  $$i = f_S C(v_{IN} - v_{OUT})$$



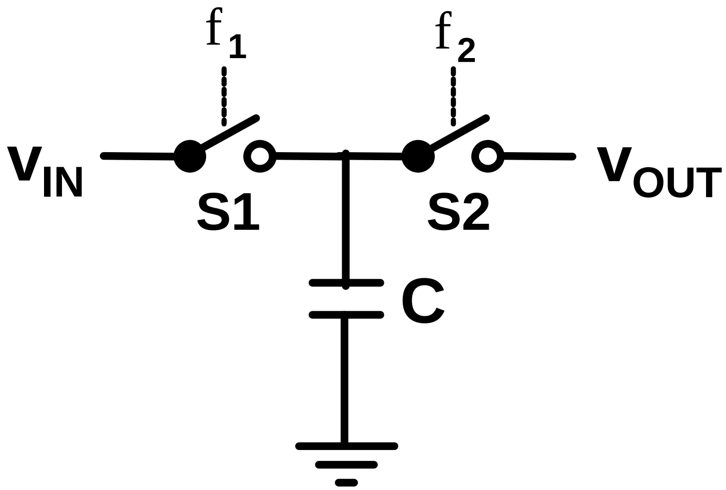
Switched-Capacitor Resistors

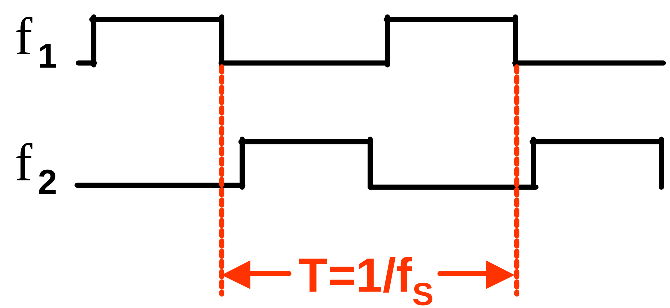
$$i = f_S C(v_{IN} - v_{OUT})$$

With the current through the switched capacitor resistor proportional to the voltage across it, the equivalent “switched capacitor resistance” is:

$$R_{EQ} = \frac{1}{f_S C}$$

Of course this current flows in “bursts”—think of “big electrons”.





Switched-Capacitor Filter

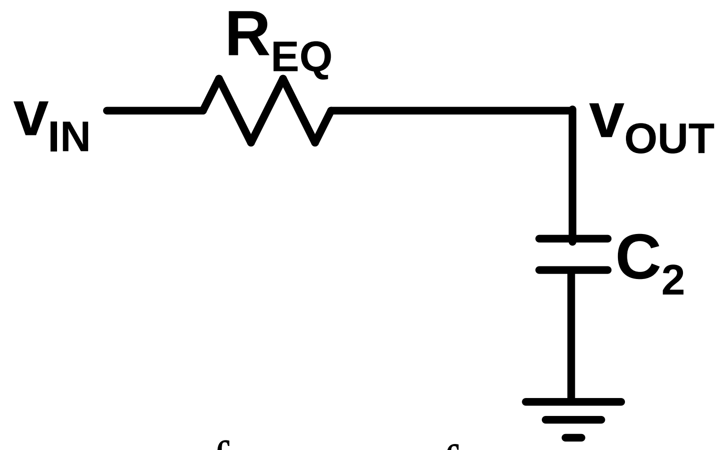
- Let's build an "SC" filter …

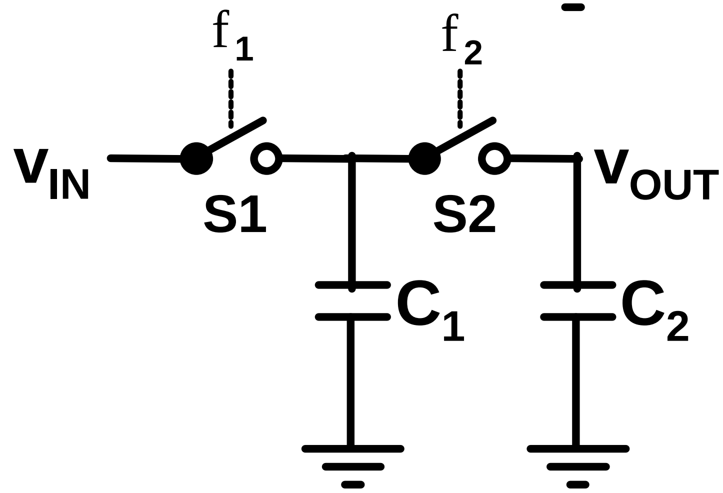
- We'll start with a simple RC LPF

- Replace the physical resistor by an equivalent SC resistor

- 3-dB bandwidth:

$$\omega_0 = \frac{1}{R_{EQ}C_2} = f_S \frac{C_1}{C_2}$$



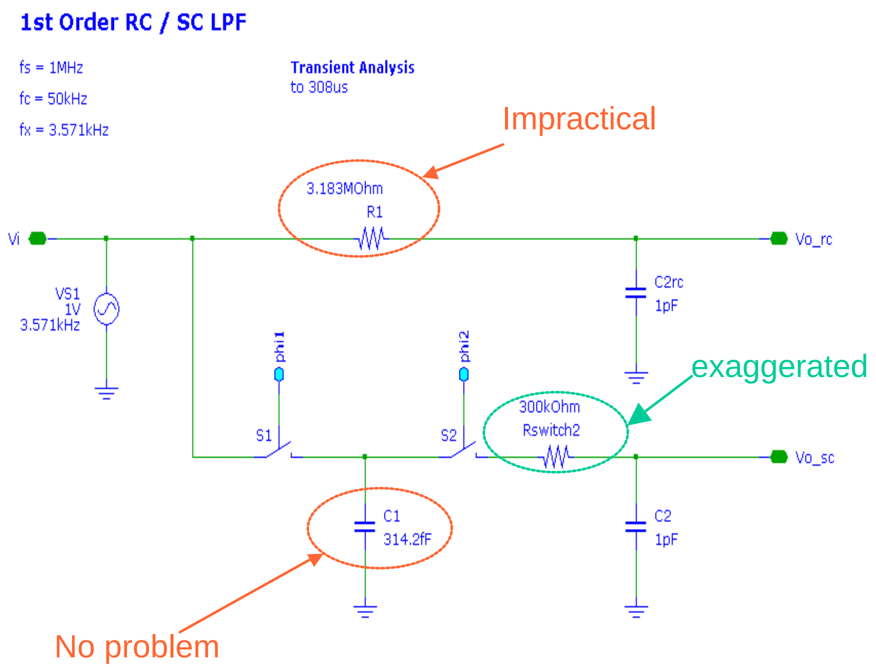


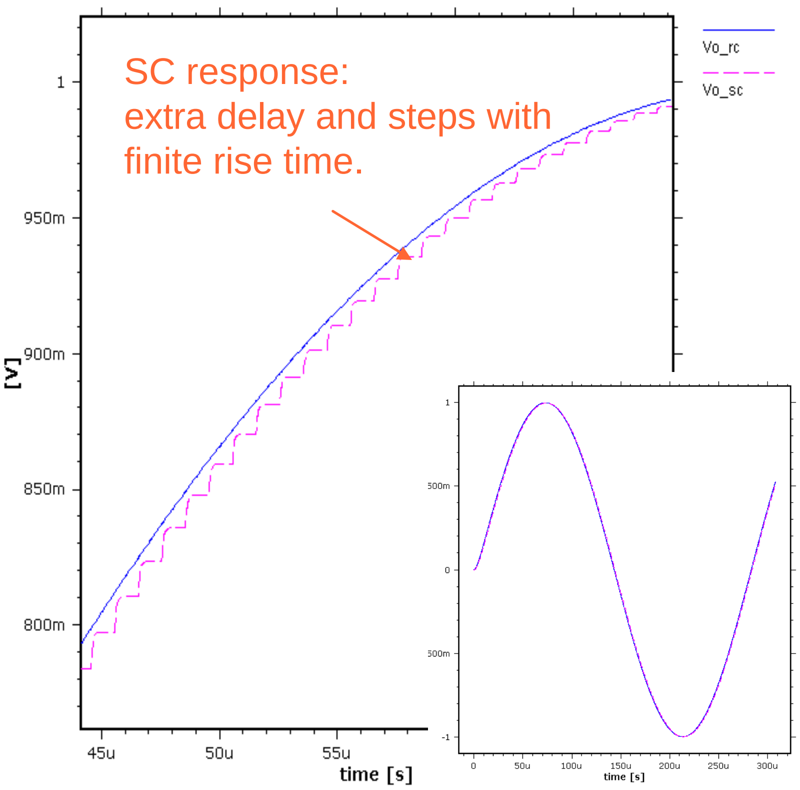
Switched-Capacitor Filters

- In SCFs, all critical frequencies track the sampling frequency
  - Crystal oscillators for f_S are stable to ~10ppm/°C
  - RC products used in active-RC filters can be tuned, but RCs in active-RC filters don't track together nearly as well

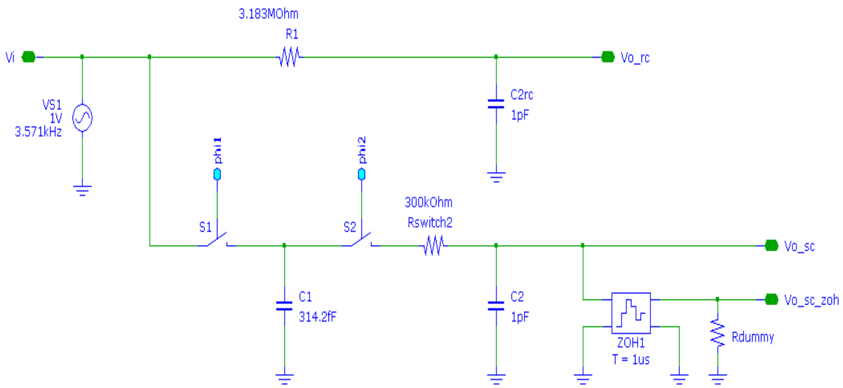
- Capacitor <u>ratios</u> in monolithic filters are perfectly stable over time and temperature
  - Capacitor ratios can't be trimmed easily
  - The trick is to achieve initial ratio accuracies of ~1000ppm out of double-poly CMOS processes

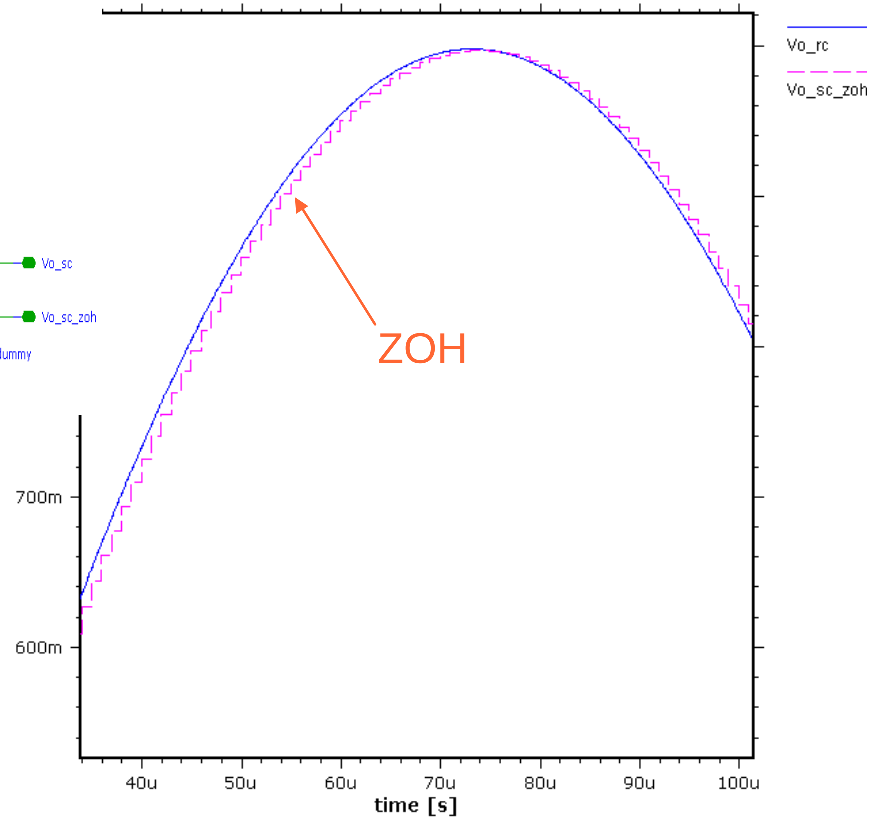
Transient Analysis





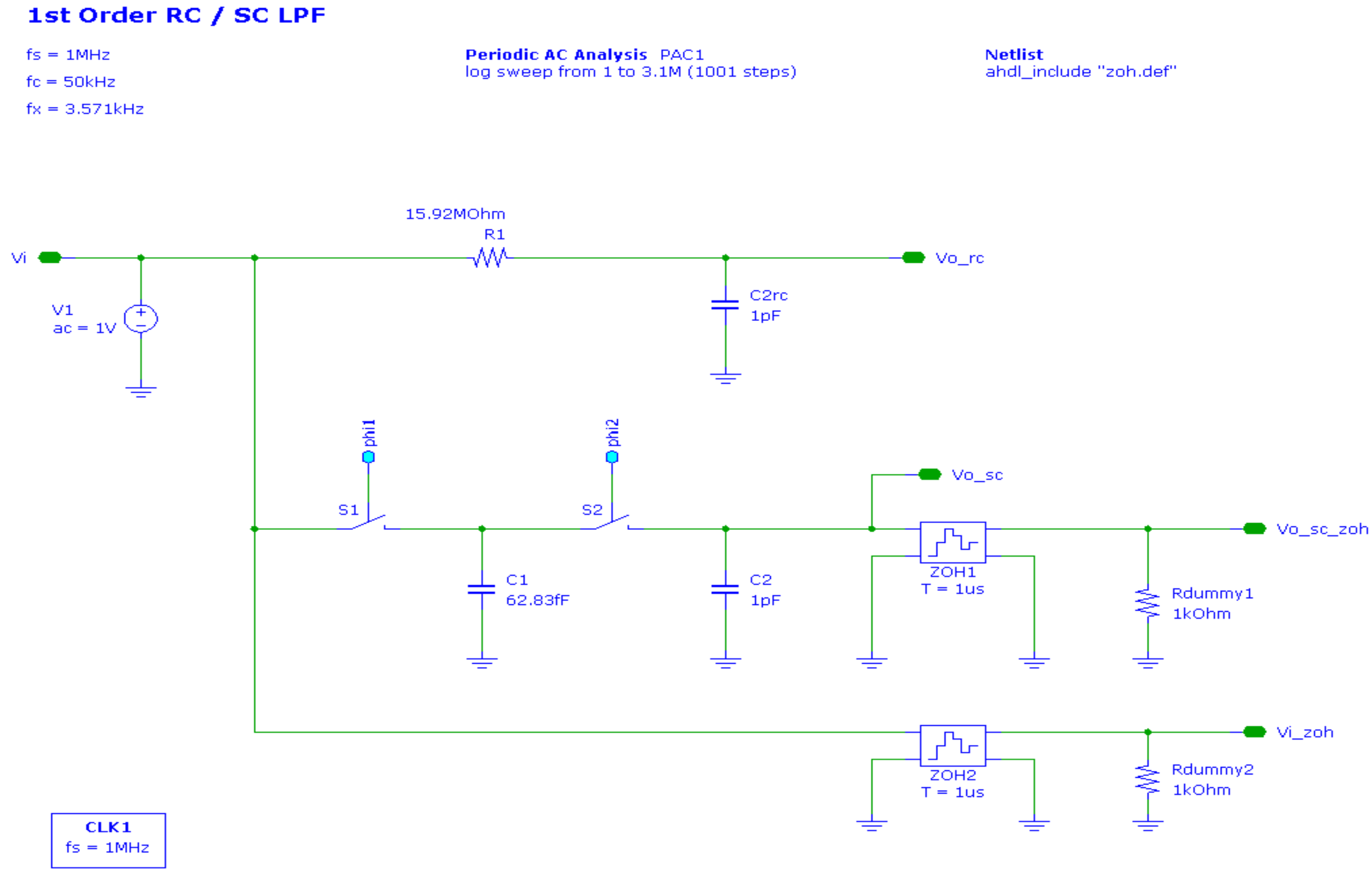
Transient Analysis



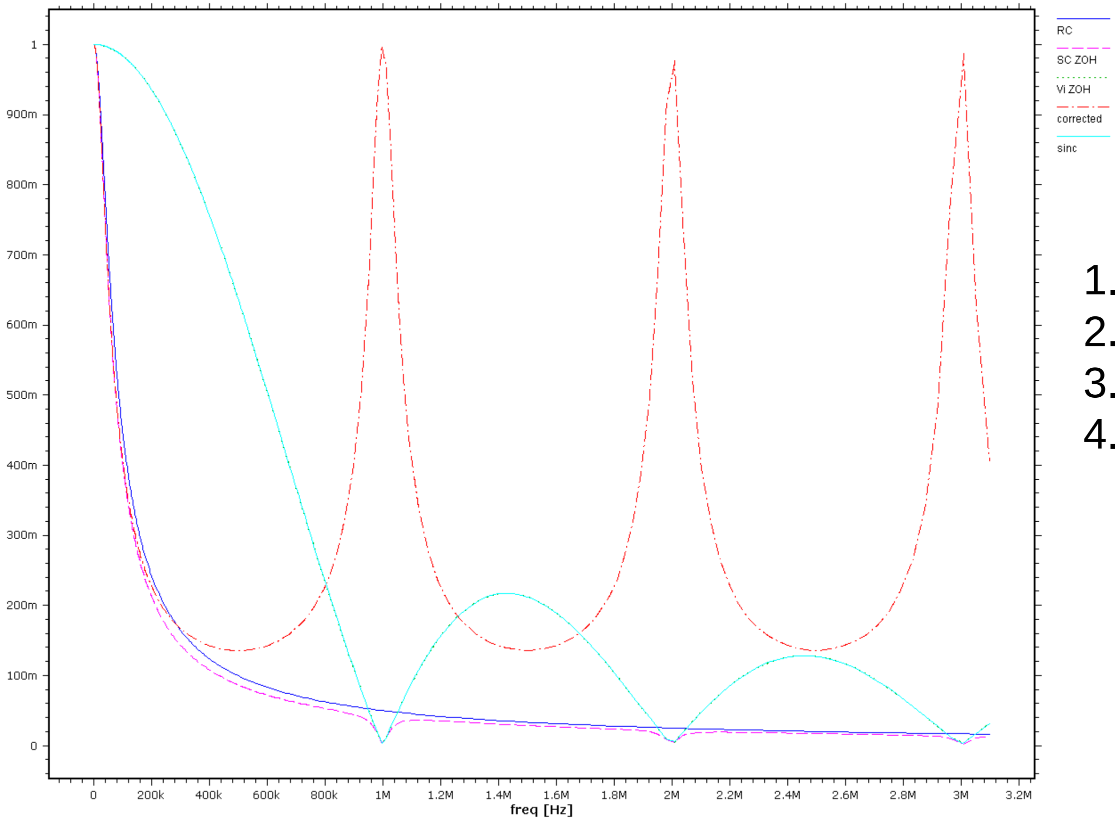


- ZOH: pick signal after settling (usually at end of clock phase)
- Adds delay and sin(x)/x distortion
- When in doubt, use a ZOH in periodic ac simulations

Periodic AC Analysis



Magnitude Response



1. RC filter output
2. SC output after ZOH
3. Input after ZOH
4. Corrected output
    - (2) over (3)
    - periodic with f_s
    - Identical to RC for $f \ll f_s/2$

Periodic AC Analysis

- SPICE frequency analysis
  - ac linear, time-invariant circuits
  - pac linear, time-variant circuits
- SpectreRF statements

```
V1 ( Vi 0 ) vsource type=dc dc=0 mag=1 pacmag=1
PSS1 pss period=1u errpreset=conservative
PAC1 pac start=1 stop=1M lin=1001
```

- Output
  - Divide results by sinc(f/f_s) to correct for ZOH distortion

Spectre Circuit File

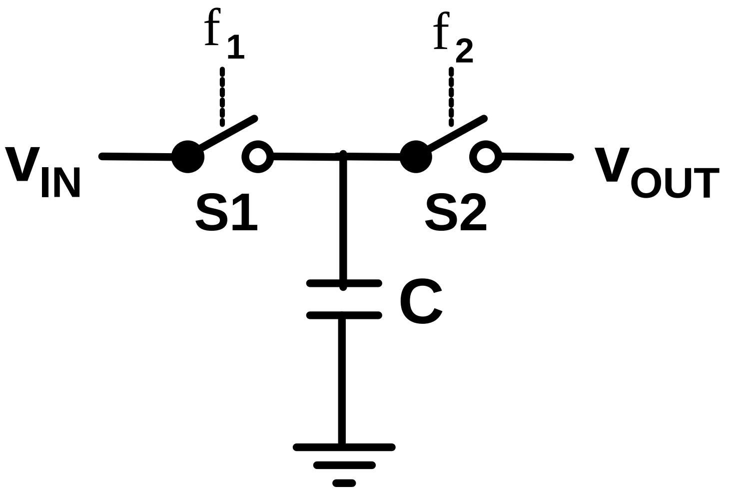
```
rc_pac
simulator lang=spectre
ahdl_include "zoh.def"

S1 ( Vi c1 phi1 0 ) relay ropen=100G rclosed=1 vt1=-500m vt2=500m
S2 ( c1 Vo_sc phi2 0 ) relay ropen=100G rclosed=1 vt1=-500m vt2=500m
C1 ( c1 0 ) capacitor c=314.159f
C2 ( Vo_sc 0 ) capacitor c=1p
R1 ( Vi Vo_rc ) resistor r=3.1831M
C2rc ( Vo_rc 0 ) capacitor c=1p
CLK1_Vphi1 ( phi1 0 ) vsource type=pulse val0=-1 val1=1 period=1u
                          width=450n delay=50n rise=10n fall=10n
CLK1_Vphi2 ( phi2 0 ) vsource type=pulse val0=-1 val1=1 period=1u
                          width=450n delay=550n rise=10n fall=10n
V1 ( Vi 0 ) vsource type=dc dc=0 mag=1 pacmag=1
PSS1 pss period=1u errpreset=conservative
PAC1 pac start=1 stop=3.1M log=1001
ZOH1 ( Vo_sc_zoh 0 Vo_sc 0 ) zoh period=1u delay=500n aperture=1n tc=10p
ZOH2 ( Vi_zoh 0 Vi 0 ) zoh period=1u delay=0 aperture=1n tc=10p
```

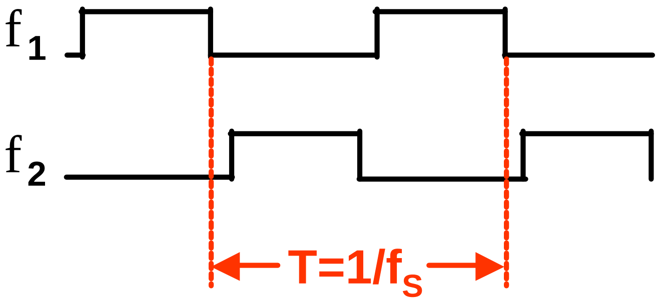
ZOH Circuit File

```
// Copy from the SpectreRF Primer

module zoh (Pout, Nout, Pin, Nin) (period, delay,
     aperture, tc)

node [V,I] Pin, Nin, Pout, Nout;
parameter real period=1 from (0:inf);
parameter real delay=0 from [0:inf);
parameter real aperture=1/100 from (0:inf);
parameter real tc=1/500 from (0:inf);
{
integer n; real start, stop;
node [V,I] hold;
  analog {
    // determine the point when aperture begins
    n = ($time() - delay + aperture) / period +
     0.5;
    start = n*period + delay - aperture;
    $break_point(start);

    // determine the time when aperture ends
    n = ($time() - delay) / period + 0.5;
    stop = n*period + delay;
    $break_point(stop);

    // Implement switch with effective series
    // resistence of 1 Ohm
    if ( ($time() > start) && ($time() <= stop))
      I(hold) <- V(hold) - V(Pin, Nin);
    else
      I(hold) <- 1.0e-12 * (V(hold) - V(Pin, Nin));

    // Implement capacitor with an effective
    // capacitance of tc
    I(hold) <- tc * dot(V(hold));

    // Buffer output
    V(Pout, Nout) <- V(hold);

    // Control time step tightly during
    // aperture and loosely otherwise
    if (($time() >= start) && ($time() <= stop))
      $bound_step(tc);
    else
      $bound_step(period/5);
  }
}
```

Switched-Capacitor Noise

- The resistance of switch S1 produces a noise voltage on C with variance kT/C
- The corresponding noise charge is $Q^2=C^2V^2=kTC$
- This charge is sampled when S_1 opens

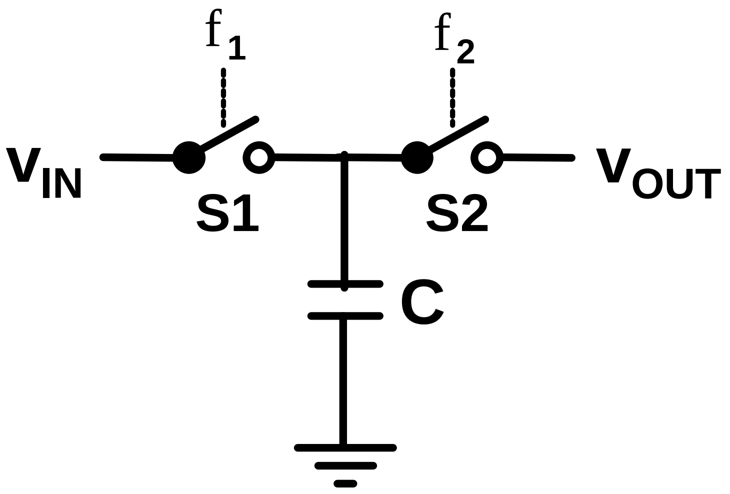


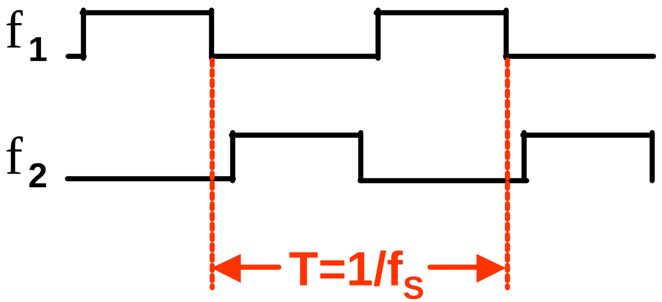


Switched-Capacitor Noise

- The resistance of switch S2 contributes to an uncorrelated noise charge on C at the end of ϕ_2

- The mean-squared noise charge transferred from v_{IN} to v_{OUT} each sample period is $Q^2=2kTC$





Switched-Capacitor Noise

- The mean-squared noise current due to S1 and S2's kT/C noise is :

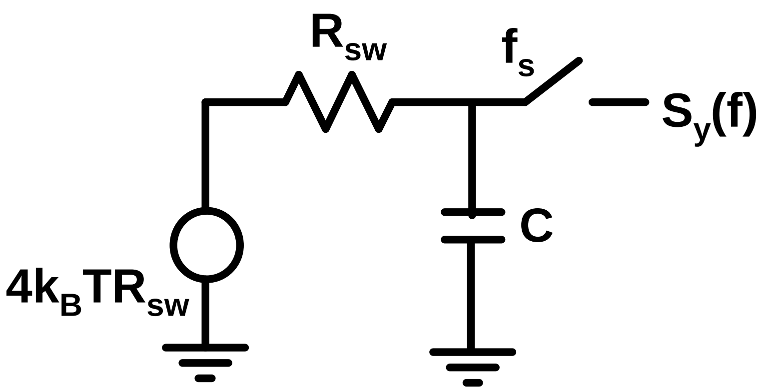
$$\overline{i^2} = \left(Qf_s\right)^2 = 2k_B TCf_s^{\,2}$$

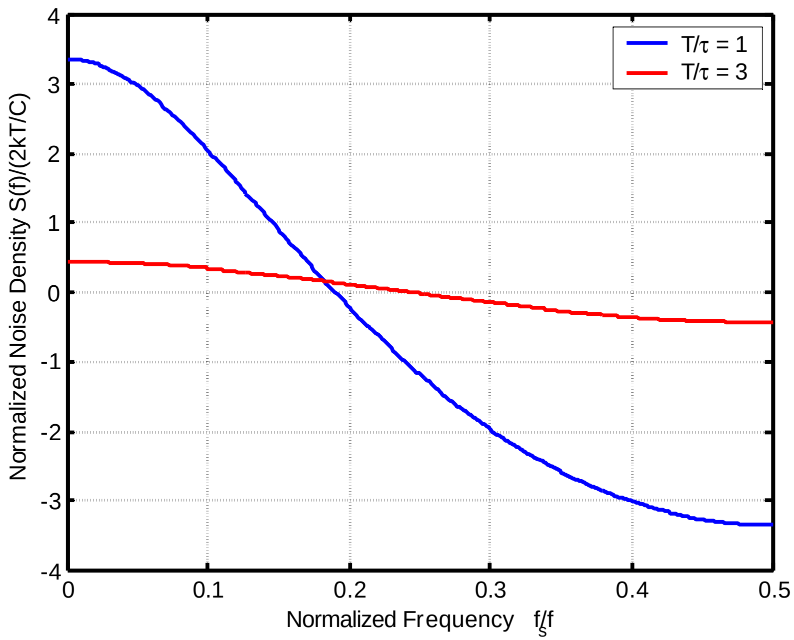
- This noise is approximately white (see next slide) and distributed between 0 and $f_s/2$ (noise spectra are single sided by convention) The spectral density of the noise is:

$$\frac{\overline{i^2}}{\Delta f} = \frac{2k_B TCf_s^{\,2}}{f_s/2} = 4k_B TCf_s = \frac{4k_B T}{R_{EQ}} \qquad \text{using} \qquad R_{EQ} = \frac{1}{f_s C}$$

- **The noise from an SC resistor equals the noise from a physical resistor with the same value!**

SC Resistor Noise Spectrum





$$S_y(f) = \frac{k_B T_r}{C} \frac{2}{f_s} \frac{1-e^{-2a}}{1+e^{-2a}\left(1-\cos 2pfT\right)}$$

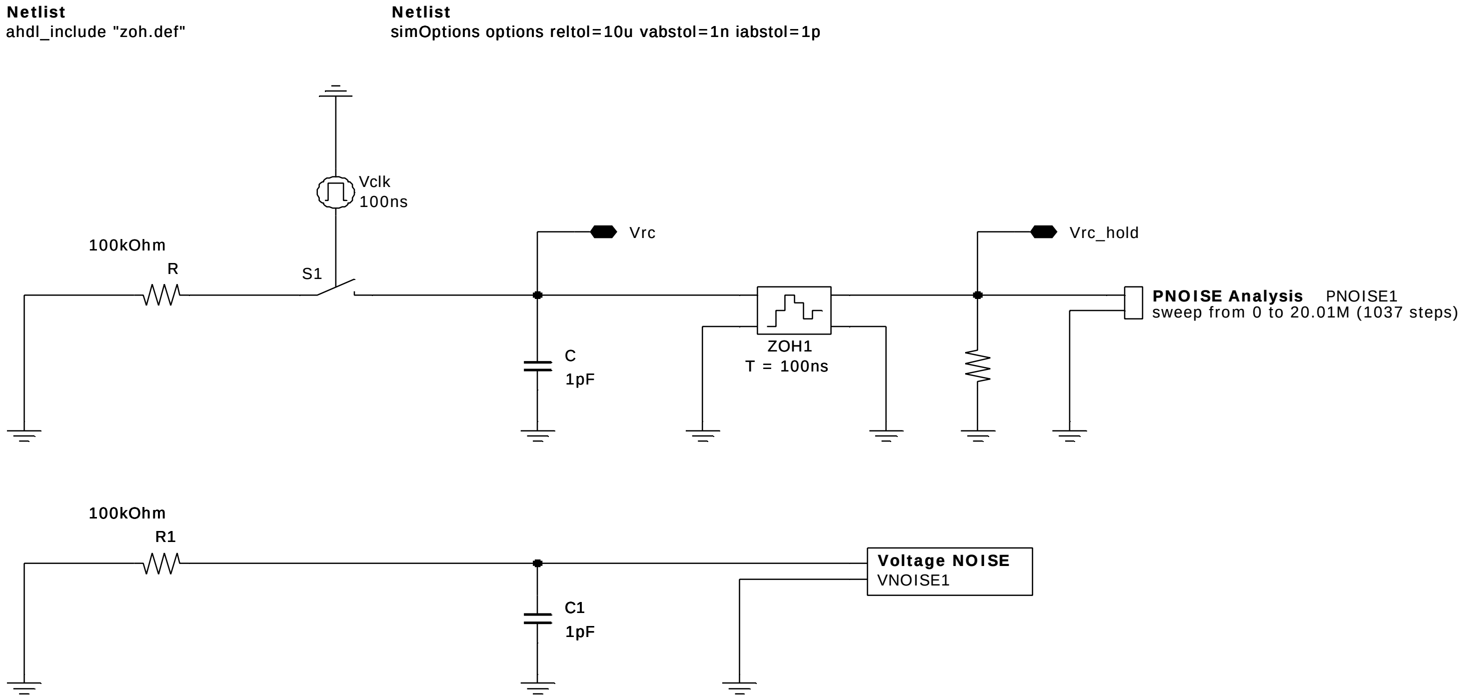
$$a = \frac{T}{R_{sw}C} \qquad \text{and} \qquad T = \frac{1}{f_s}$$

$$\int_0^{f_s/2} S_y(f)df = \frac{k_B T_r}{C}$$

- Noise essentially white for T/t > 3
- Settling constraints ensure that this condition is usually met in practice
- Note: This is the noise density of an SC resistor only. The noise density from an SC filter is usually not white.

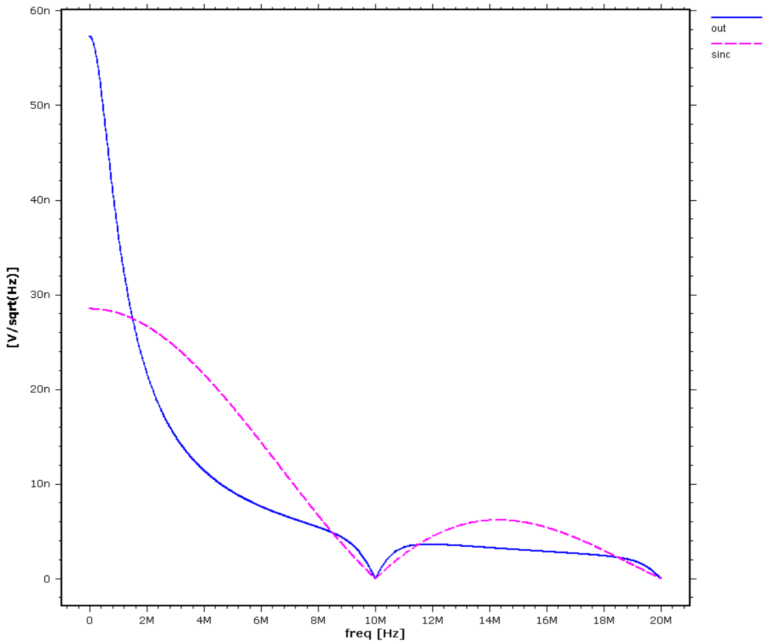
Periodic Noise Analysis

Sampling Noise from SC S / H

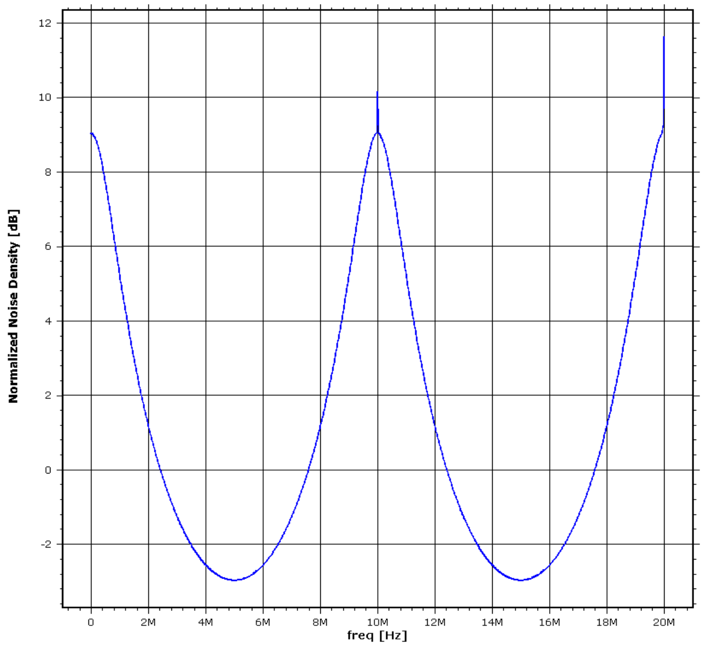



```
PSS pss period=100n maxacfreq=1.5G errpreset=conservative
PNOISE ( Vrc_hold 0 ) pnoise start=0 stop=20M lin=500 maxsideband=10
```

Sampled Noise Spectrum

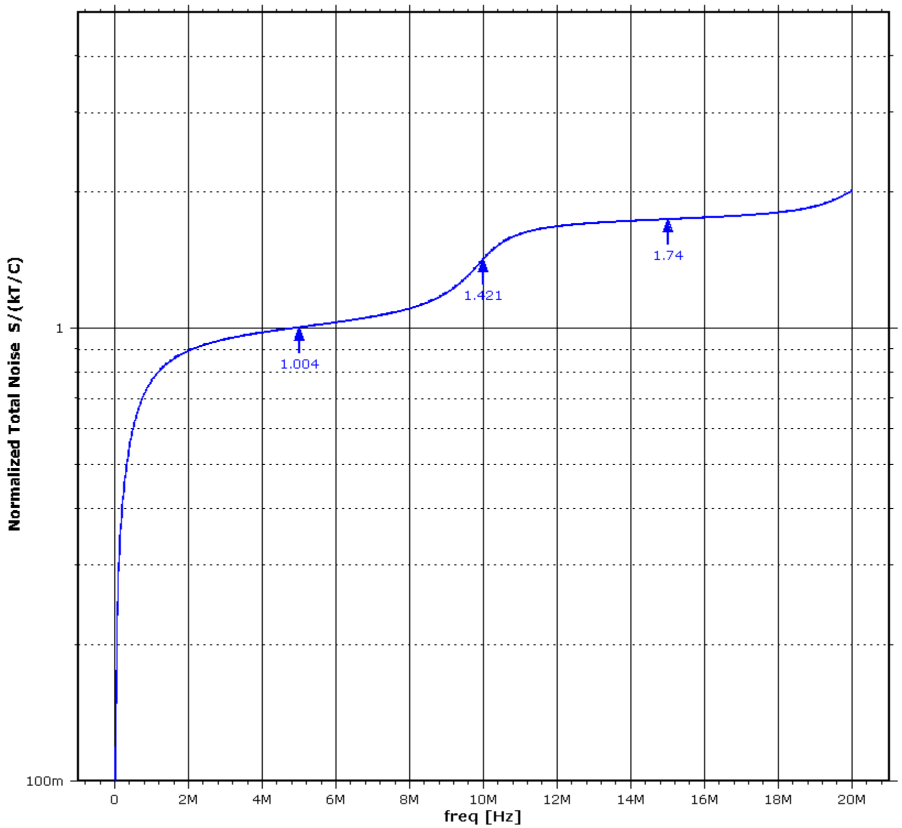


Density of sampled noise with sinc distortion.



Normalized density of sampled noise, corrected for sinc distortion.

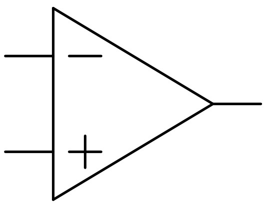
Total Noise

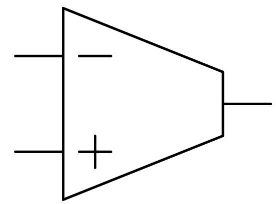


Sampled noise in $0 \ldots f_s/2$: $62.2\mu V$ rms

(expect $64\mu V$ for 1pF)

Opamps versus OTA





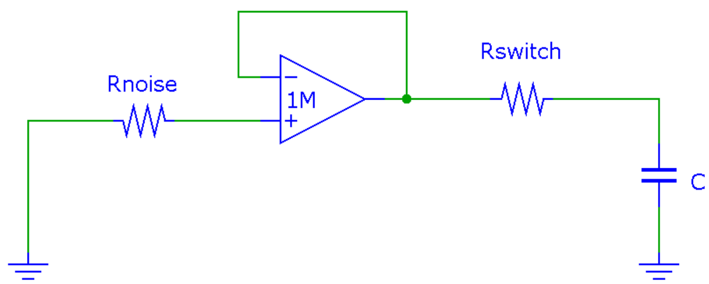
- Low impedance output
- Can drive R-loads
- Good for RC filters,
  OK for SC filters

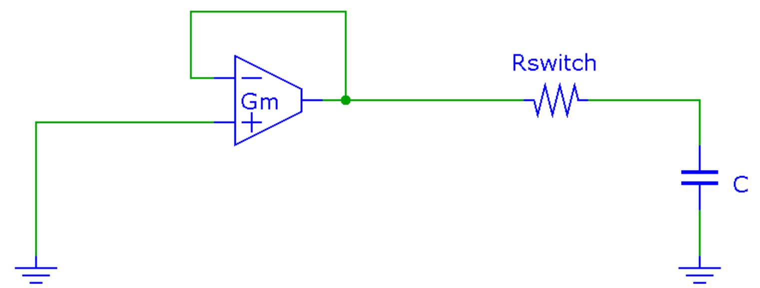
- Extra buffer adds complexity,
  power dissipation

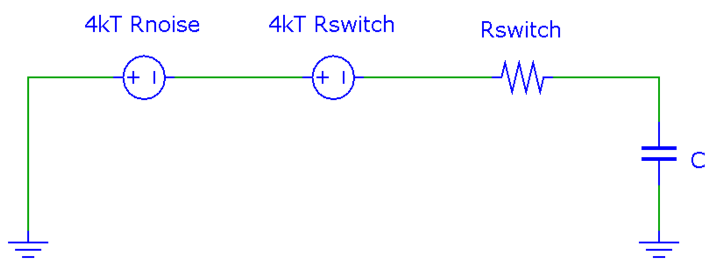
- High impedance output
- Cannot drive R-loads
- Ideal for SC filters

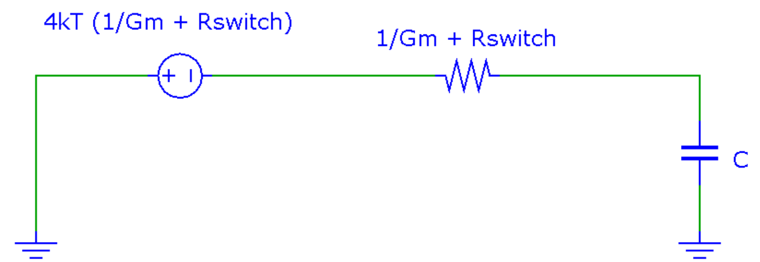
- Simpler than Opamp

Opamps versus OTA Noise









$$\sqrt{\overline{v_{oT}^2}} = \frac{kT}{C}\left(1 + \frac{R_{\text{noise}}}{\text{R}_{\text{switch}}}\right)$$

$$\sqrt{\overline{v_{oT}^2}} = \frac{kT}{C}$$

Opamp and switch noise add

OTA contributes no excess noise
(actual designs can increase noise)

Amplifier Bandwidth Requirements

SC Filter:

$$t \le \frac{T}{10} = \frac{1}{10 f_s} \qquad \text{for } 16^{+} \text{ Bit settling accuracy}$$

$$t = \frac{1}{w_u} = \frac{1}{2p f_u}$$

$$\rightarrow f_u \ge \frac{10}{2p} f_s \cong 2 f_s$$

$$f_s = 8 \ldots 100 \times f_{\text{corner}}$$

$$f_u = 16 \ldots 200 \times f_{\text{corner}}$$

CT Filter:

$$f_u = 50 \ldots 1000 \times f_{\text{corner}}$$

→ SC filters have comparable or slower amplifier bandwidth requirements than CT filters

SC Filter Summary

- ✓ Pole and zero frequencies proportional to
  - – Sampling frequency f_s
  - – Capacitor ratios
  - ➢ High accuracy and stability in response
  - ➢ Low time constants realizable without large R, C
- ✓ Compatible with transconductance amplifiers
  - – "No" excess opamp noise
  - – Reduced circuit complexity, power
- ✓ Amplifier bandwidth requirements comparable to CT filters
- o Catch: Sampled data filter → aliasing