Discrete Fourier Transform

The DFT of a block of N time samples

$\{a_n\} = \{a_0, a_1, a_2, \ldots, a_{N-1}\}$

is a set of N frequency bins

$\{A_m\} = \{A_0, A_1, A_2, \ldots, A_{N-1}\}$

where:

$$\mathbf{A_m = \sum_{n=0}^{N-1} a_n W_N^{mn}} \qquad \mathbf{m = 0,1,2,\ldots,N-1}$$

$$\mathbf{W_N \equiv e^{j2\pi/N}}$$

Inverse DFT

The inverse DFT flips a set of frequency bins

$\{A_m\} = \{A_0, A_1, A_2, \ldots, A_{N-1}\}$

back to the corresponding time domain samples

$\{a_n\} = \{a_0, a_1, a_2, \ldots, a_{N-1}\}$:

$$\mathbf{a_n = \frac{1}{N} \sum_{m=0}^{N-1} A_m W_N^{-mn}} \qquad \mathbf{n=0,1,2,\ldots,N-1}$$

$$\mathbf{W_N \equiv e^{j2\pi/N}}$$

DFT Properties

- DFT of N samples spaced $T=1/f_s$ seconds:
  - N bins
  - Bin m represents frequencies at $m * f_s/N$ [Hz]

- DFT resolution:
  - Proportional to $1/(NT)$ in [Hz/bin]
  - NT is total time spent gathering samples

DFT Properties

$$A_{N-m} = A_m^*$$

- Given that A_{N-m} and A_m are complex conjugates,

½$\mathbf{A_{N-m}}$½= ½$\mathbf{A_m}$½

- For real time sequences, frequency domain magnitudes mirror around $f_S/2$

Energy Theorem

Relates energy in the time domain to energy in the frequency domain:

$$\sum_{n=0}^{N-1} |a_n|^2 = \frac{1}{N} \sum_{m=0}^{N-1} |A_m|^2$$

Energy Theorem

$$\sum_{n=0}^{N-1} |a_n|^2 = \frac{1}{N} \sum_{m=0}^{N-1} |A_m|^2$$

$$\sqrt{\frac{1}{N} \sum_{n=0}^{N-1} |a_n|^2} = \sqrt{\frac{1}{N^2} \sum_{m=0}^{N-1} |A_m|^2}$$

$$a_{rms} = \frac{1}{N} \sqrt{\sum_{m=0}^{N-1} |A_m|^2}$$

Energy Theorem

- A <u>dc sequence</u> $a_n = a_{dc}$ has only one nonzero term in its DFT:

$$\mathbf{A_0 = N a_{dc}}$$

- A <u>sinewave</u> at the center frequency of bin m has 2 nonzero terms, A_m and A_{N-m}, and their magnitudes are the same:

$$\mathbf{a_{rms} = \frac{\sqrt{2}}{N}|A_m|}$$

$$\mathbf{|A_m| = \frac{N a_{rms}}{\sqrt{2}}}$$

The Energy Theorem

- <u>White noise</u> with rms value a_{rms} distributes its energy evenly into all frequency bins
  - If the magnitude in each bin is A,

$$\mathbf{a_{rms} = \frac{1}{N}\sqrt{NA^2} = \frac{A}{\sqrt{N}}}$$

$$\mathbf{A = a_{rms}\sqrt{N}}$$

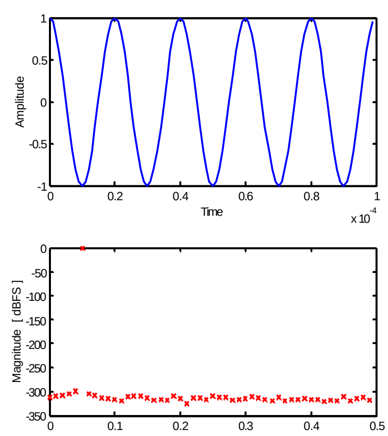
DFT Magnitude Plots

- Because A_m magnitudes are symmetric around $f_S/2$, it is redundant to plot $|A_m|$'s for m>N/2
- Usually magnitudes are plotted on a log scale normalized so that a full scale sinewave of rms value a_{FS} yields a peak bin of 0dBFS:

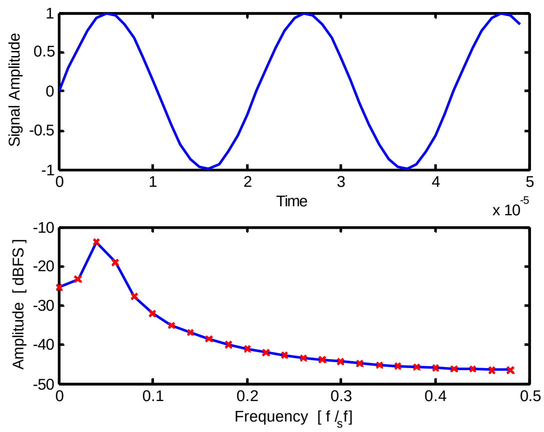
$$|A_m| \text{ (dBFS)} = 20 \log_{10} \frac{|A_m|}{a_{FS}\, N/2}$$

Normalized DFT

```
fs  = 1e6;
fx  = 50e3;
afs = 1;
N   = 100;

% time vector
t = linspace(0, (N-1)/fs, N);
% signal
y = afs * cos(2*pi*fx*t);
% spectrum
s = 20 * log10(abs(fft(y)/N/afs*2));
% drop redundant half
s = s(1:N/2);
% frequency vector (normalized to fs)
f = (0:length(s)-1) / N;
```



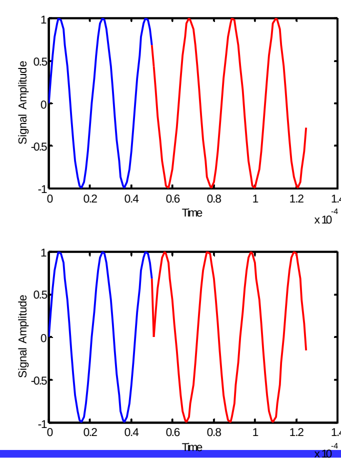
“Another” Example …



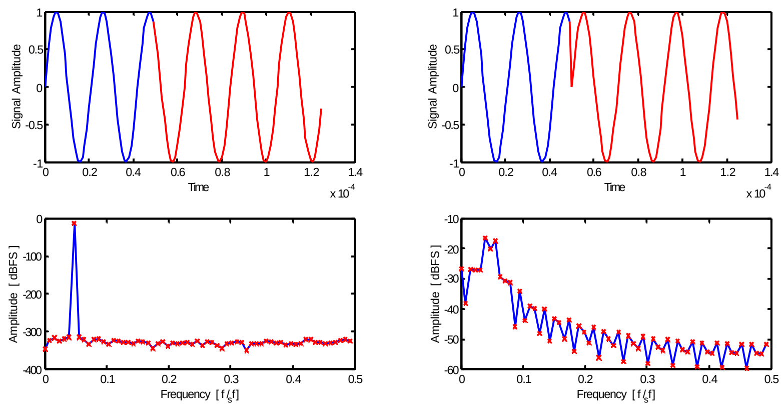
This does not look like the spectrum of a sinusoid …

DFT Periodicity

- The DFT implicitly assumes that time sample blocks repeat every N samples
- With a non-integral number of periods periods within our observation window, the input yields a huge amplitude/phase discontinuity at the block boundary
- This energy spreads into all frequency bins as “spectral leakage”
- Spectral leakage can be eliminated by either
  - An integral number of sinusoids in each block
  - Windowing

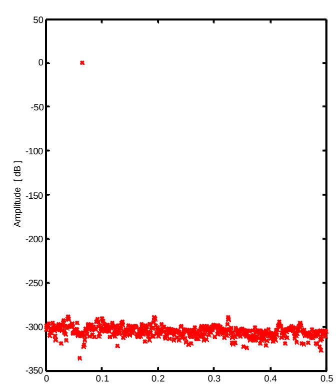


Spectra



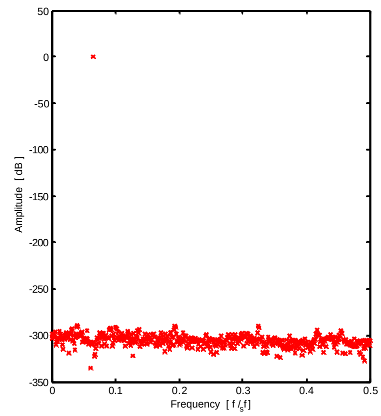
Integral Number of Periods

```
fs = 1e6;

% number of full cycles in test
cycles = 67;

% power of 2 speeeds up analysis
% but make N/cycles non-integer!
N = 2^10;

% signal frequency
fx = fs*cycles/N
```



Integral Number of Periods

- Fundamental falls into a single DFT bin
- Noise (here numerical quantization) occupies all other bins
- “Integral number of periods” constrains signal frequency f_x
- Alternative: windowing →

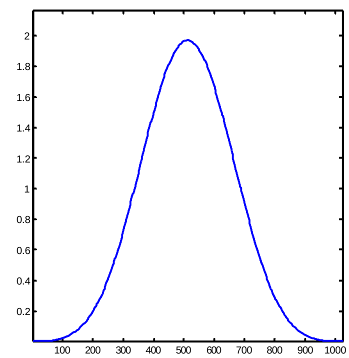


Windowing

- Spectral leakage can also be virtually eliminated by “windowing” time samples prior to the DFT
  - Windows taper smoothly down to zero at the beginning and the end of the observation window
  - Time samples are multiplied by window coefficients on a sample-by-sample basis
- Windowing sinewaves places the window spectrum at the sinewave frequency
  - Convolution in frequency

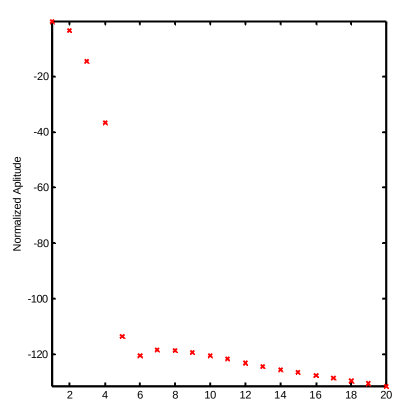
Nuttall Window

- Time samples are multiplied by window coefficients on a sample-by-sample basis
- Multiplication in the time domain corresponds to convolution in the frequency domain



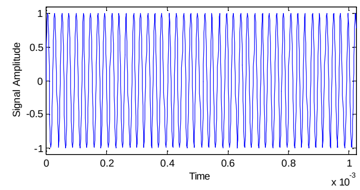
DFT of Nuttall Window

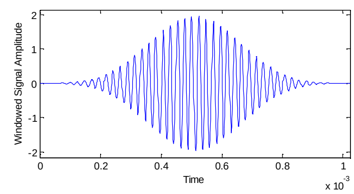
- Only first 20 bins shown
- Response essentially zero for bins > 5



Windowed Data

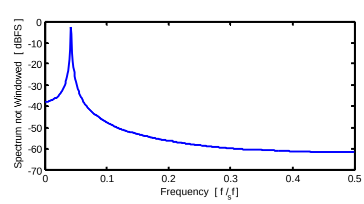
- The plot on the right shows the signal before and after windowing
- Windowing removes the discontinuity at block boundaries

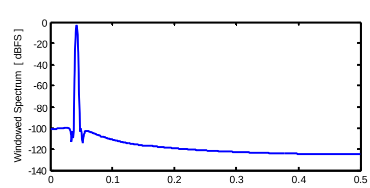




DFT of Windowed Signal

- Spectra of signal before and after windowing
- Window gives ~ 100dB attenuation of sidelobes (use longer window for higher attenuation)
- Signal energy "smeared" over several (approximately 10) bins





Integral Cycles versus Windowing

- Integral number of cycles
  - Signal energy falls into single DFT bin
  - Requires careful choice of f_x
  - Ideal for simulations, usually impractical for measurements
- Windowing
  - No restrictions on f_x and no need to lock it to f_s
    → ideal for measurements
  - Signal energy (and harmonics) distributed over several DFT bins
  - Requires more datapoints for set accuracy

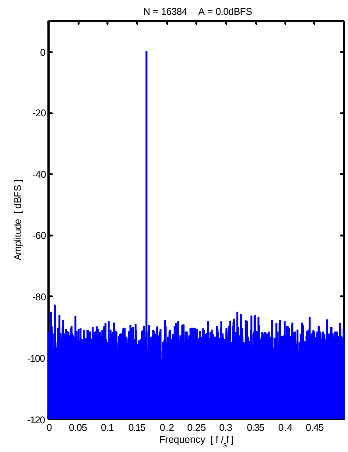
Spectral ADC Testing

- ADC with B bits
- ±1 full scale input

```
B = 10;
delta = 2/(2^B-1);
th = -1+delta/2:delta:1-delta/2;
x = sin(…);
y = adc(x, th) * delta - 1;
```

ADC Output Spectrum

- Signal amplitude:
  - Bin: `N * fx/fs + 1` (Matlab arrays start at 1)
  - A = 0dBFS
- SNR?

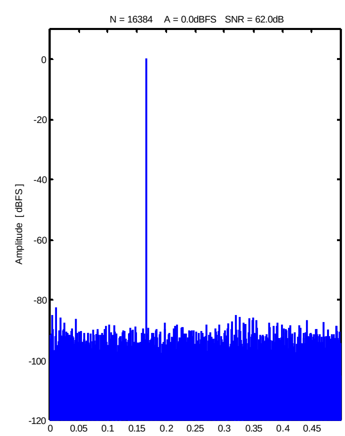


ADC Output Spectrum

- Noise bins: all except signal bin

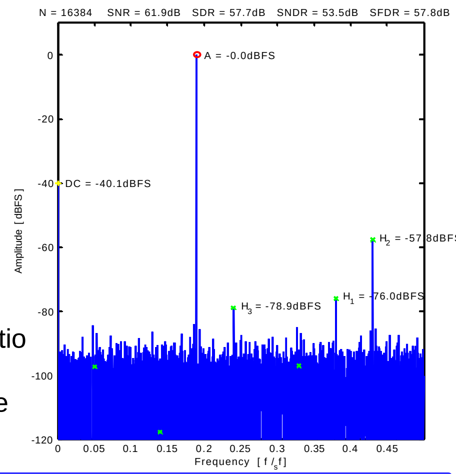
```
bx = N*fx/fs + 1;
As = 20*log10(s(bx))
sn(bx) = 0;
An = 10*log10(sum(sn.^2))
SNR = As - An
```

- SNR = 62dB (10 bits)



Spectral Components

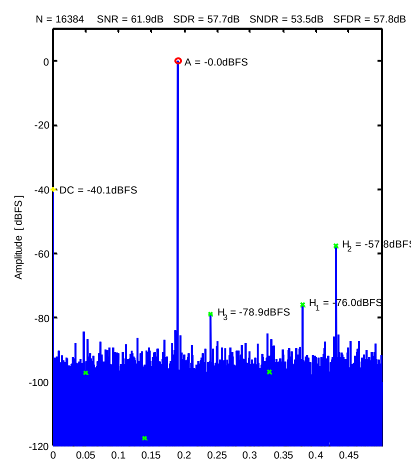
- Signal S
- DC
- Distortion D
- Noise N



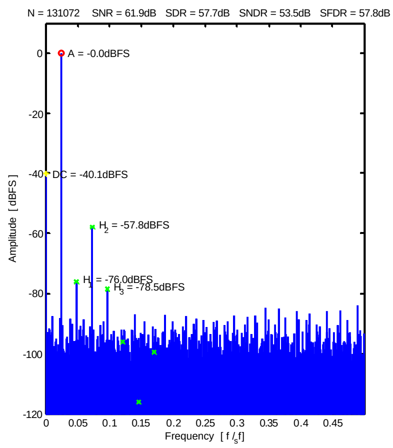
- Signal-to-noise ratio
  SNR = S / N
- Signal-to-distortion ratio
  SDR = S / D
- Signal-to-noise+distortion ratio
  SNDR = S / (N+D)
- Spurious-free dynamic range
  SFDR

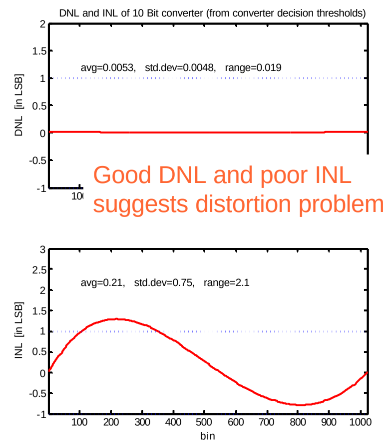
Distortion Components

- At multiples of f_x
- Aliasing:
  - $f_0 = f_x = 0.18\ f_s$
  - $f_1 = 2\ f_0 = 0.36\ f_s$
  - $f_2 = 3\ f_0 = 0.54\ f_s \rightarrow 0.46\ f_s$
  - $f_3 = 4\ f_0 = 0.72\ f_s \rightarrow 0.28\ f_s$
  - $f_4 = 5\ f_0 = 0.90\ f_s \rightarrow 0.10\ f_s$
  - $f_5 = 6\ f_0 = 1.08\ f_s \rightarrow 0.08\ f_s$



Spectrum versus INL, DNL

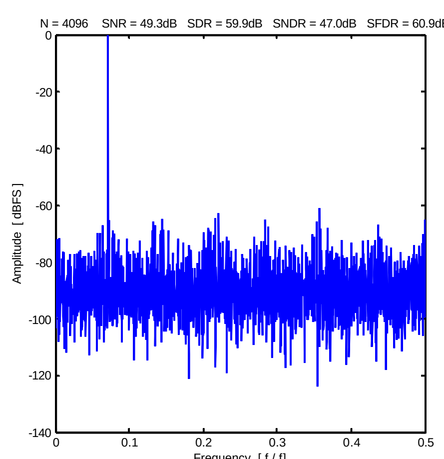




Good DNL and poor INL suggests distortion problem

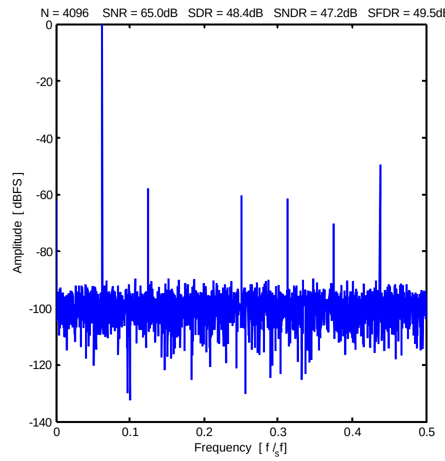
Noise

- At right is the spectrum of a 10-Bit converter
- SNDR = 47dB – something's amiss
- Distortion?
  SDR = 59.9dB – no
- Must be a noise problem, but is it thermal or quantization noise?



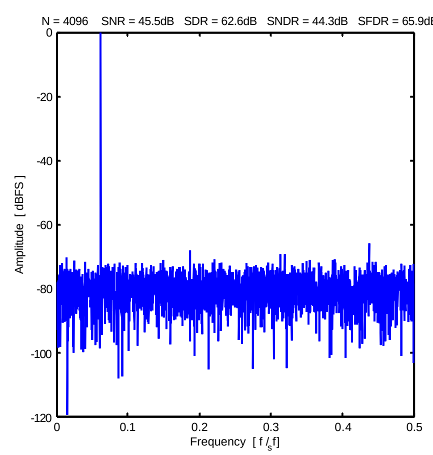
Noise

- At right is the spectrum of a same 10-Bit converter for $f_x = f_s / 16$
- Since f_x divides f_s, the quantization noise is periodic!
- It falls into the same bins the harmonics would normally occupy
- Hence
  - SNR → thermal noise
  - SDR → quantization noise (apparently the culprit)



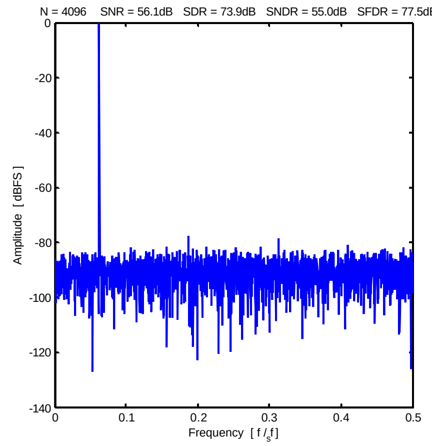
Noise

- Same converter again ($f_x = f_s / 16$)
- The quantization noise problem has been fixed: SDR = 62dB … for 10bits Congratulations!
- But apparently the fix causes a thermal noise problem: SNR = 45.5dB
- Another revision (our competitors like this …)



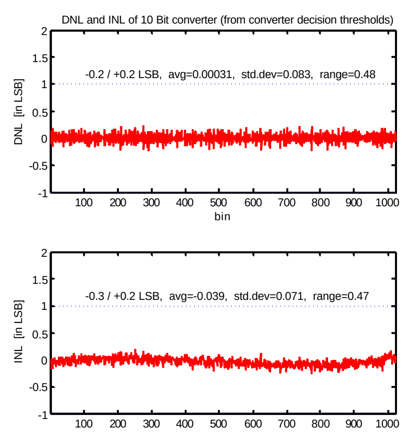
Noise

- One more time …
  ($f_x = f_s / 16$)
- The quantization noise is not a major error:
  SDR = 74dB
- SNR = 56.1dB
  This corresponds to Gaussian noise with variance Δ/2 at the converter input … a reasonable design choice



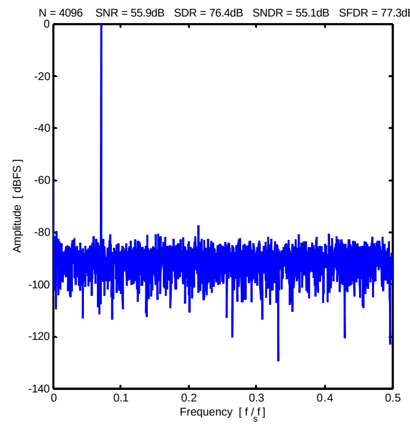
Noise

- The DNL and INL confirm the good result
- But the INL shows some “bowing” … let’s see if our test masked a distortion problem



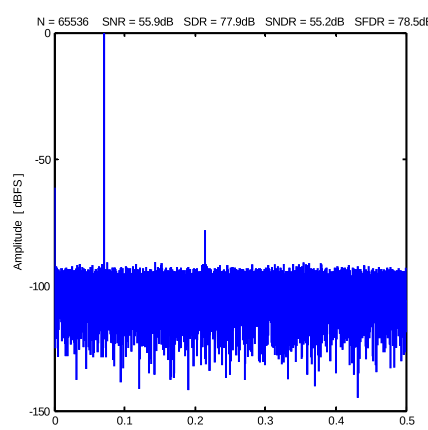
Noise

- For that we revert to simulating with f_s/f_x non-integer
- A 3rd harmonic is barely visible
- How can we "lift" it out of the noise?



Noise

- Increasing N, the number of samples (and hence the measurement or simulation time) distributes the noise over more bins
- More bins … less noise power per bin (total noise stays same)
- SFDR = 78dB for 10Bit is acceptable in many applications (e.g. digital imaging)



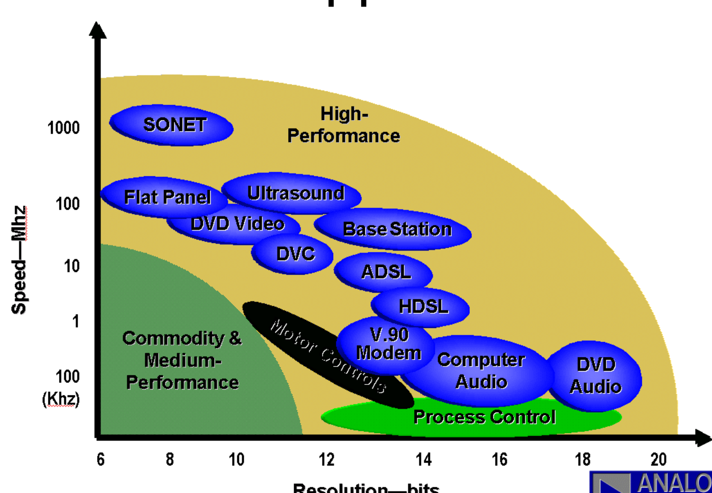
Effective Number of Bits

- Is a 10-Bit converter with 55dB SNDR really a 10-Bit converter?

- Effective Number of Bits

$$ENOB = \frac{SNDR - 1.76\text{dB}}{6.02\text{dB}}$$
$$= \frac{55 - 1.76}{6.02} = 8.8\text{Bits}$$

ADC Applications



ANALOG DEVICES

Example: AD9235 12Bit / 65MS/s

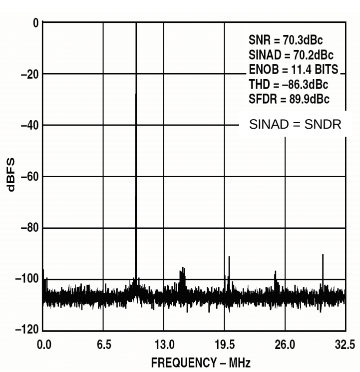
FEATURES

- Single 3 V Supply Operation (2.7 V to 3.6 V)
- SNR = 70 dBc (to Nyquist)
- SFDR = 90 dBc (to Nyquist)
- Low Power: 300 mW at 65 MSPS
- Differential Input with 500 MHz Bandwidth
- On-Chip Reference and SHA
- DNL = ±0.4 LSB
- Flexible Analog Input: 1 V p-p to 2 V p-p Range
- Offset Binary or Two's Complement Data Format
- Clock Duty Cycle Stabilizer

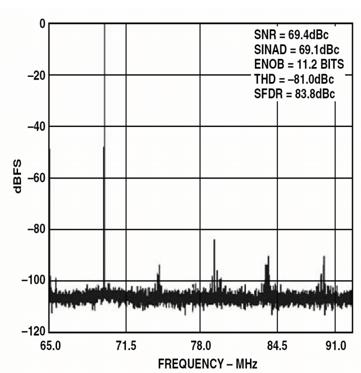
APPLICATIONS

- Ultrasound Equipment
- IF Sampling in Communications Receivers:
  IS-95, CDMA-One, IMT-2000
- Battery-Powered Instruments
- Hand-Held Scopemeters
- Low Cost Digital Oscilloscopes

AD9235 Spectra

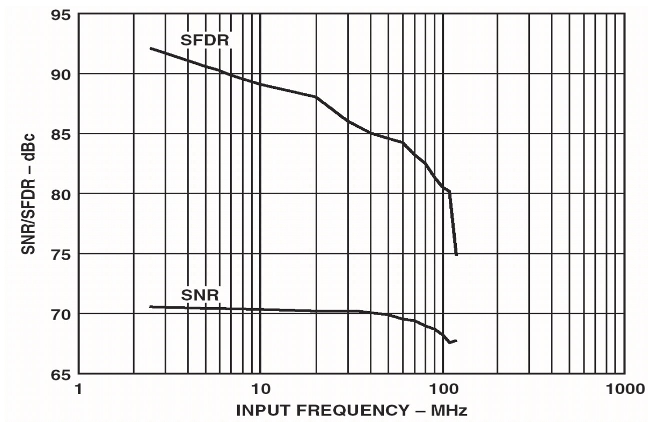


TPC 1. Single Tone 8K FFT with f_{IN} = 10 MHz

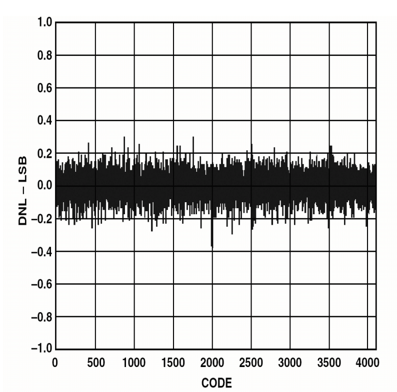


TPC 2. Single Tone 8K FFT with f_{IN} = 70 MHz

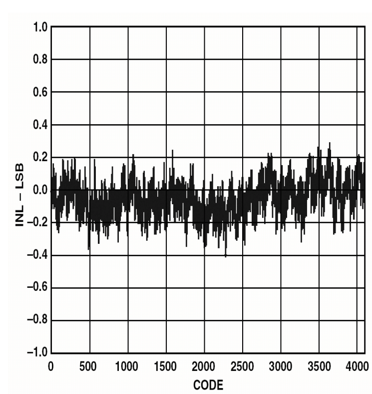
AD9235 SNR / SFDR



AD9235 DNL / INL

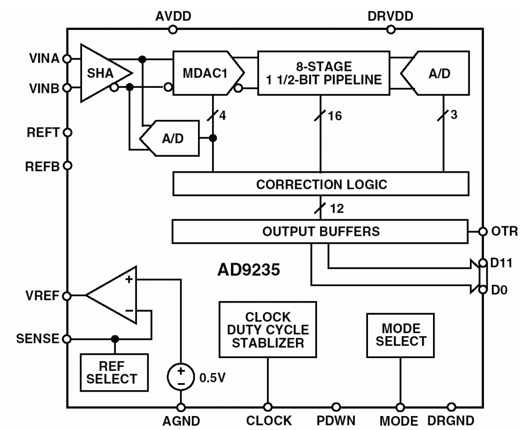


TPC 24. Typical DNL



TPC 23. Typical INL

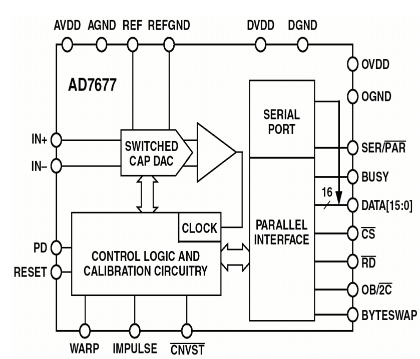
AD9235 Block Diagram



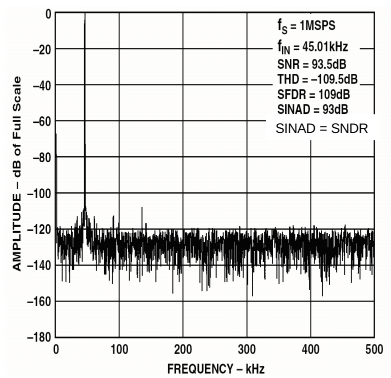
AD7677 16Bit / 1MS/s

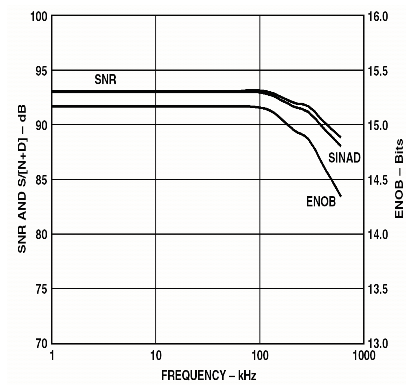
FEATURES
Throughput: 1 MSPS
INL: ±1 LSB Max (±0.0015% of Full Scale)
16 Bits Resolution with No Missing Codes
S/(N+D): 94 dB Typ @ 45 kHz
THD: –110 dB Typ @ 45 kHz
Differential Input Range: ±2.5 V
Both AC and DC Specifications
No Pipeline Delay
Parallel (8 Bits/16 Bits) and Serial 5 V/3 V Interface
Single 5 V Supply Operation
115 mW Typical Power Dissipation, 15 µW @ 100 SPS
Power-Down Mode: 7 µW Max
Packages: 48-Lead Quad Flatpack (LQFP)
48-Lead Frame Chip Scale (LFCSP)
Pin-to-Pin Compatible Upgrade of the AD7664/AD7675/
AD7676

APPLICATIONS
CT Scanners
Data Acquisition
Instrumentation
Spectrum Analysis
Medical Instruments
Battery-Powered Systems
Process Control

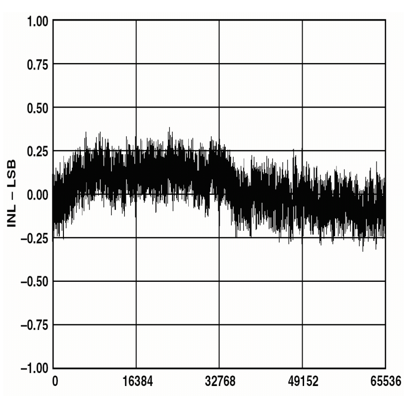


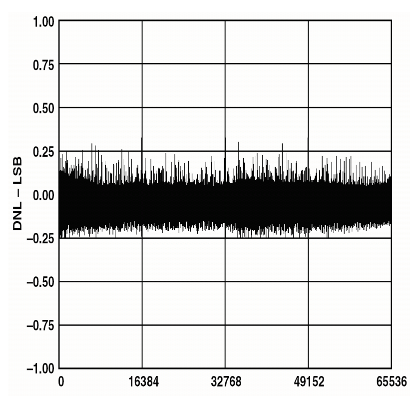
AD7677 Spectrum



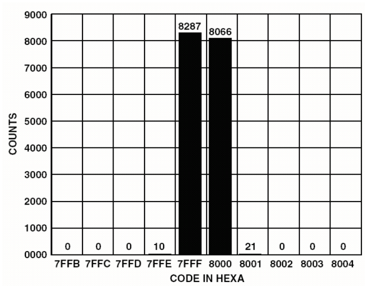


AD7677 DNL / INL

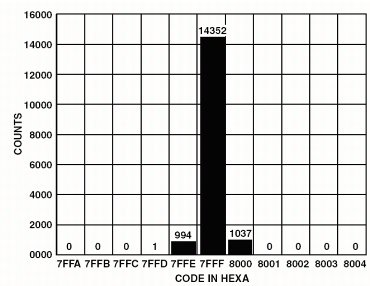




AD7677 DC Input



TPC 2. Histogram of 16,384 Conversions of a DC Input at the Code Transition



TPC 5. Histogram of 16,384 Conversions of a DC Input at the Code Center

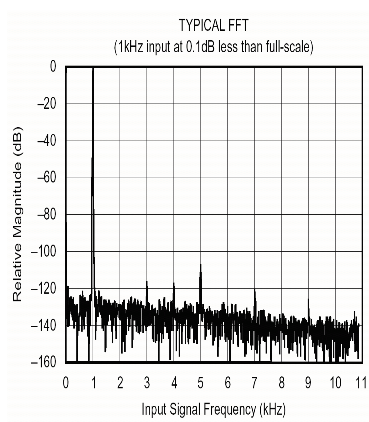
ADS1254 24Bits / 20kS/s

FEATURES

- 24 BITS—NO MISSING CODES
- 19 BITS EFFECTIVE RESOLUTION UP TO 20kHz DATA RATE
- LOW NOISE: 1.8ppm
- FOUR DIFFERENTIAL INPUTS
- INL: 15ppm (max)
- EXTERNAL REFERENCE (0.5V to 5V)
- POWER-DOWN MODE
- SYNC MODE
- LOW POWER: 4mW at 20kHz
- SEPARATE DIGITAL INTERFACE SUPPLY 1.8V to 3.6V

APPLICATIONS

- CARDIAC DIAGNOSTICS
- DIRECT THERMOCOUPLE INTERFACES
- BLOOD ANALYSIS
- INFRARED PYROMETERS
- LIQUID/GAS CHROMATOGRAPHY
- PRECISION PROCESS CONTROL



Recent Nyquist ADC Performance:

ISSCC Nyquist ADCs

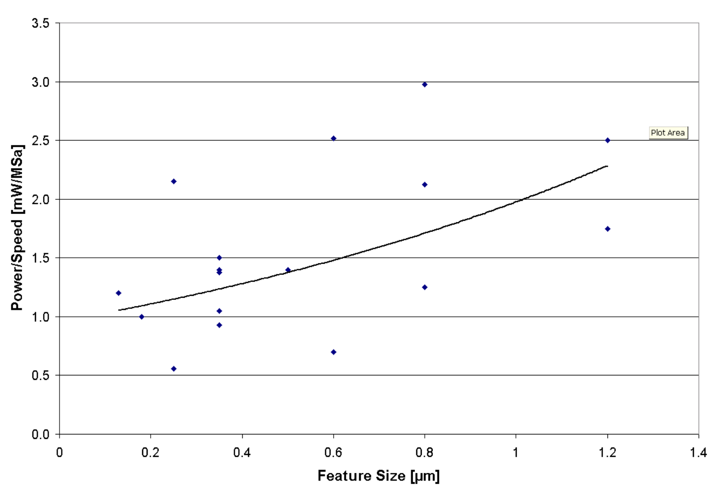
Year	paper #	lead author	power(mW)	res	SNDR(db)	Fs	sig BW	FOM
2002	10.2	Scholtens	340	6	32dB	1600	660	77
2001	8.1	Choi	545	6	33dB	1300	650	54
2001	8.2	Geelen	300	6	36dB	900	450	93
2000	26.2	Sushihara	400	6	32dB	800	200	20
2000	26.1	Nagaraj	187	6	35.2dB	700	250	76
1999	18.5	Tamba	400	6	35dB	500	250	35
1999	18.6	Yoon	330	6	33dB	500	75	10
2002	18.2	Lin	0.48	6	33dB	22	11	1023
2002	10.3	Sushihara	50	7	36.7dB	450	225	308
2002	10.1	Poulton	4600	8	38.5dB	4000	2000	37
2000	2.5	Ming	250	8	46dB	80	20	16
2002	10.4	Jamal	234	10	57dB	120	60	182
2001	8.3	Park	180	10	57dB	100	50	197
1999	18.3	Hoogzaad	65	10	57dB	40	20	228
2002	10.5	Miyazaki	16	10	54dB	30	15	470
1999	18.2	vanderPloeg	195	10	58dB	25	5	20
1999	18.4	Brandt	75	10	60dB	20	10	133
2002	10.6	Kuttner	12	10	55dB	20	10	468
2000	2.3	Singer	500	12	70dB	65	32	202
2001	8.4	vanderPloeg	295	12		54	25	0
2000	2.4	Pan	850	12	64dB	50	25	47
2002	18.4	Kulhalli	30	12	68dB	21	10	837
1999	18.1	Erdogan	16	12	71dB	0.125	0.05	11
2002	18.5	Waltari	715	13		50	25	0
2000	2.2	Choe	800	13	66dB	40	20	50
2000	2.1	Moreland	1250	14	75dB	100	25	112
2001	8.5	Kelly	340	14	73dB	75	37.5	493
2001	8.6	Yu	860	14		40		0
2000	2.7	Chen	720	14	74dB	20	10	70

Figure of Merit:

$$FOM = \frac{ENOB \times BW}{P}$$

All Bandwidths are in MHz, all FOM are 10^9

10-Bit ADC Power



12-Bit ADC Power